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Abstract

Opponents of school choice sometimes charge that vouchers, charter schools , and tuition tax credits would strip funding and talented students from the public schools. Proponents say this is exactly what is needed to provide extra competition for public schools. Flight to private schools may happen if parents think private schools are good substitutes for public schools. For goods with explicit market prices, economists estimate substitutability by specifying a demand curve and finding a cross-price elasticity, but the non-market nature of schooling has prevented this. The current study finds a way to estimate the demand for public schooling and calculate a cross-price elasticity by exploiting Rosen's (1974) two-stage hedonic technique. It estimates the cross-price elasticity between public schooling and the price of private schooling to be 0.66: Americans view private schools as moderately strong substitutes for public schools. However, the flight to private schools is likely to be twice as strong in high-income school districts than in low-income school districts. The use of spatial statistics accounts for potential spillovers and omitted variable bias in the house price hedonics and the demand curve estimation. In fact, the -1.32 price elasticity of demand is much larger than the -0.20 to -0.40 estimates generally achieved by non-spatial studies.

JEL Classification codes: H41, R21, R22
Keywords: local public goods provision, private school competition, spatial statistics, education vouchers, spillovers

"This was the Super Bowl of school choice, and children won," –Clint Bolick, Vice President of the Institute for Justice, June 27, 2002

"The majority took a wrecking ball today…Vouchers are an unconscionable diversion of resources away from the public schools," –Barry W. Lynn, President of Americans United for Separation of Church and State, June 27, 2002 (Walsh, 2002)

1. Introduction

School choice programs, in the form of charter schools, school vouchers, and tax credits for private school tuition, are becoming more common in the United States. The legislative, executive, and judicial branches of government are all active contributors. For example, The No Child Left Behind Act of 2001 allows students to transfer out of failing schools and recommends converting thousands of failing public schools into charter schools. President Bush's 2003 budget proposed giving families up to $2500 per child in tax credits if they chose a private school rather than a failing public school. And in 2002 the U.S. Supreme Court ruled in *Zelman v Simmons-Harris* that the government may fund vouchers for private, religious schools. What would happen if school choice programs became widespread? Would parents flee to private schools, abandoning the public schools? The idea of 'abandoning the public schools' is broadly defined to mean less public schooling demanded. It means a smaller market share for traditional public schools. It also means less funding for public schools, as money follows students to other schools; and it might mean less frequent passage of tax levies. Abandoning the public schools might also mean greater cream-skimming, in which the most talented students leave the public schools (Epple and Romano, 1998). Less money and lower peer effects might reduce public school quality, so 'abandoning the public schools' also implies a lower quality of public schooling demanded. But whether school choice programs like vouchers and tax credits would make parents abandon public schools for private schools is an empirical question. The answer depends on the degree to which parents perceive public and private schools to be substitutes, and it is this substitutability that the current study explores.

The degree to which public and private schools are substitutes has special importance for school voucher programs. Currently in the United States tax money funds public schools, and residents who live in a public school district may send their children to their own public school for no extra charge. If residents want to send their children to a private school or a different public school district, they must continue to pay taxes for their assigned public school and additionally pay full tuition for the school their child attends. School choice programs break the link between houses and public school district assignments. One school choice program is a school voucher. If the assigned public school district spends $7,000 per pupil, one form of school voucher system would give parents a voucher worth $7,000 which may be redeemed at the assigned public school, or it may be used toward tuition at the public or private school of the parents' choice. Some polling suggests that about 60% of Americans favor private school vouchers (Walsh, 2001). The strongest supporters of vouchers are racial minorities and people living in low-income areas with poor quality public schools (Walsh, 2001; Sandy, 1992). Voucher programs have been proposed or enacted in California, Florida, Michigan, Louisiana, Maine, D.C., Colorado, Indianapolis, Milwaukee, San Antonio, Atlanta, Cleveland, Vermont, New Hampshire, New Jersey, and other states and cities throughout the United States (Merrifield, 2002).

With their increasing public awareness, it is useful to ask whether vouchers will provide competition for public schools. Stronger competitive effects will be seen the more private and public schools are seen as close substitutes. Opponents of vouchers point out that a voucher system may gut public schools, transferring students and their associated funding away from the already cash-strapped public schools toward private schools. This 'mass exodus' argument assumes that private schools are seen as good substitutes for public schools. So in order to better evaluate the likely competitive effects of vouchers and to assess the validity of the mass exodus objection to private school vouchers, it is necessary to estimate the degree to which Americans perceive public and private schools to be substitutes.

Some previous research has investigated the sensitivity of private school enrollment to tuition, and the relationship between public school expenditures and private school enrollment. These studies have provided valuable (but contradictory) insights on the degree to which public and private schooling are substitutes. However, in introductory microeconomics courses, students are taught to measure substitutes and complements from the cross-price elasticity of demand. The non-market nature of schooling has prevented researchers from estimating the cross-price elasticity of demand. But the current study exploits the theory of implicit markets (Rosen, 1974) to find the price of a unit of public and private schooling from the housing market. It can then estimate the demand for public schooling to find the cross-price elasticity between public and private schooling, to see to what extent Americans view public and private schools as substitutes. If private schools are strong substitutes for public schools, school choice programs may induce an exodus of students and money from public schools to private schools, and scare public schools into providing higher quality service at a lower price. If private schools are weak substitutes for public schools, school choice programs might subsidize private—and often religious—schools, increase taxes[1], and fail to provide much additional competition for public schools.

The empirical estimation consists first of estimating a series of house price hedonic regressions, then using the results from the first stage to estimate the demand for public schooling. The estimation technique, spatial statistics, addresses identification and spatial dependence, which may arise from spillovers and omitted variables. Private schooling appears to be something of a substitute for public schooling. The cross-price elasticity of demand is 0.66. Furthermore, the cross-price elasticity of demand is stronger for higher-income public school districts. For people in the 75th percentile of house prices, the cross-price elasticity of demand is 0.82; it is 0.43 for people in the 25th percentile. The results suggest that the public schools more likely to be gutted by school choice programs are the ones with the highest incomes. By the same token, low-income schools are the ones with the least incentive to respond to the additional competition that school choice programs provide, which suggests that means-tested voucher programs aren't likely to have much effect.

In fact, the weaker cross-price elasticity for low-income households may help explain the low take-up rate on some school choice programs. Most school choice programs are restricted to students in low-performing schools, which are predominantly low-income. The relatively low cross-price elasticity of demand for schooling by poor households found in the current study may help explain why only 1097 out of 270,000 eligible students transferred from failing schools in Chicago last year under the No Child Left Behind Act, and why only 215 transferred out of 204,000 eligible in Los Angeles (Helfand and Rubin, 2004).

2. The Price of a Unit of Education

Previous studies have estimated the cross-price elasticity of demand for competition between public and private universities (Allen and Shen, 1999), but not between public and private elementary and secondary schools. Previous studies have also investigated the price elasticity of demand for private schooling (Chiswick and Koutromanes, 1996; Hoenack, 1997). Others investigate the effect public school expenditures has on private school enrollment (Hamilton and Macauley, 1991; Goldhaber, 1999), and the effect of private school enrollment on public school expenditures (Erekson, 1982; Goldhaber, 1999). Buddin, et al. (1998) regress a family's decision to use private schools as a function of both private school tuition and public school proficiency test scores. But while these studies all provide valuable information about the determinants of the demand for public and private schooling, none has calculated a cross-price elasticity between public and private schooling. There is a reason they have not.

A demand function contains the price of the good, the price of related goods, and other demand shifters. Because there is no explicit market, there is no readily observable unit price of public schooling. There is also no readily observable unit price of private schooling. This lack of data is probably what has kept researchers from calculating a cross-price elasticity of demand. Despite the lack of explicit prices, the *implicit* price of public and private schooling may be inferred from the housing market using the hedonic approach, and these implicit prices may be used to estimate a demand curve (Rosen, 1974).

The hedonic approach explains the price of a complex good like a house as the sum of the prices of its components, including the value of the rooms, central air conditioning, and the attractiveness of public schools and private school alternatives. There are consumers and producers of housing. Consumers of housing bid for levels of housing characteristics. Bids differ because consumers have different tastes and different family size. Producers of housing maximize profit by offering different combinations of housing characteristics. Firms differ in size and specialization, so their offer curves differ. Consumers and producers match up in the market for housing characteristics, and tangencies between the bid and offer curves yield implicit price schedules for the housing characteristics. Rosen (p. 40) points out that his implicit market framework is particularly well-suited to the analysis of local public goods like schooling, and many studies have used Rosen's technique to find the capitalization of and demand for non-market goods like environmental quality and schooling services.

Public schools and the attractiveness of private school alternatives provide a stream of value to homeowners, either directly through usage or as an option value. The prices of public and private schooling derived from the house price hedonic equate consumers' marginal rates of substitution with producers' marginal rates of transformations; therefore, the resulting implicit prices are based on willingness to pay and marginal costs, as revealed through the housing market. As such, the implicit prices of non-market goods "play the same role here as do direct observations on prices in the standard theory" (Rosen, 1974, p.50). The two-stage house price hedonic framework has long been used to calculate the implicit price of non-market goods in the public, urban, and environmental economic literatures. The current study applies Rosen's two-stage hedonic approach to the calculation of the cross-price elasticity between public and private schooling, providing a new way to assess the degree to which public and private schooling are substitutes. The first stage of Rosen's two-stage hedonic demand technique is estimating a house price hedonic.

3. First Stage: The Need for Six House Price Hedonics

The hedonic method expresses the price of a house as the summation of the value of its characteristics. These characteristics include physical attributes such as the number of rooms and the square footage of the house and the yard. Other less tangible attributes also affect house price. These include crime in the community, pollution levels, noise, and the attractiveness of the local schools.

The current study follows Rosen's (1974) two-stage hedonic approach, but the main shortcoming of Rosen's method is that the estimated marginal attribute prices may not provide any real information over what the original hedonic provided. The only new information is the functional form restriction placed on the demand equation. If there is no new information, the coefficients of the second-stage demand equation are not identified from the first-stage hedonic (Tinbergen, 1956; Brown and H. Rosen, 1982). Although there are single-equation methods to address identification (e.g., Quigley, 1982; Ekeland, et al., 2002), Brown and Rosen's (1982) market segmentation approach is the most widely accepted solution in the literature. According to Ekeland, et al. (2002, p. 307), "If preferences are stable and the distributions of preferences across markets are stable, but technologies are different for exogenous reasons, then multimarket variation [Brown and Rosen's approach] shifts the hedonic function against stable preferences and identifies preference parameters."

Therefore, this study derives marginal prices following Brown and Rosen's suggested method of segmenting the sample by housing market to deal with the identification problem. A separate hedonic house price function is estimated for each of the six major urban areas in Ohio: Akron, Cleveland, Cincinnati, Columbus, Dayton, and Toledo. Estimating each urban area separately results in six different schooling coefficients. The six different coefficients are used to calculate the implicit prices, which are then pooled and estimated in a single demand equation to achieve identification. All previous segmentation studies use urban areas to segment their housing markets, and they commonly use four to seven urban areas (e.g., Palmquist, 1984; Brasington, 2003; Zabel and Kiel, 2000).

4. The Use of Spatial Statistics to Estimate the Six House Price Hedonics

Consider the traditional least squares hedonic estimation given by Equation (1), where v is house value and X is the matrix of explanatory variables:

$$v = X\beta + \varepsilon, \;\; \varepsilon \sim N(0,\sigma^2) \tag{1}$$

House price hedonic regressions with individual sale prices such as Equation (1) tend not to be statistically independent. In fact, tests for statistical independence often show spatial autocorrelation in the residuals. Spatial autocorrelation is expected for at least three reasons. One, nearby houses have similar prices. Two, neighbors' behavior can affect a house's value directly or through subdivision associations. And three, non-housing determinants of house value are not fully captured by the variables included in the hedonic regressions (LeSage 1997, 2000). Estimating Equation (1) with ordinary least squares does not account for spatial dependence between observations, which may lead to biased, inefficient and inconsistent parameter estimates (Anselin, 1988, p. 58-59).

The spatial Durbin model can address the problem of spatial dependence (Pace and Barry, 1997a). The spatial Durbin model includes a spatial lag of the dependent variable as well as spatial lags of the explanatory variables in X, as in Equation (2):

$$v = \rho Wv + X\beta + W\underline{X}\alpha + \varepsilon, \; \varepsilon \sim N(0,\sigma^2 I_n) \tag{2}$$

In Equation (2) the scalar term ρ is the spatial autoregressive parameter to be estimated. It measures the degree of spatial dependence between the values of nearby houses in the sample. The W term is an n by n spatial weight matrix. It has non-zero entries in the i,jth position, reflecting houses that are nearest neighbors to each of the i homes in the sample. The current study initially selects the five nearest neighbors to form the matrix W. Therefore each row of W contains five non-zero entries to represent the five houses in the sample that are closest to house i. In this manner the spatial weight matrix W summarizes the spatial configuration of the houses in the sample. Next, $\underline{X}$ is the explanatory variable matrix X with the intercept excluded, and α is the parameter associated with the spatial lag of the explanatory variables.

The ρWv term in (2) captures the extent to which the price of each house is related to the price of neighboring houses. Such spatial interplay is appropriate because, among other reasons, when a house is put on the market, the offer price is often set with the knowledge of the selling price of similar houses in the neighborhood. Multiple listing services publish offer prices and newspapers in Ohio publish sale prices, so typically offers and bids on houses will be influenced by offers and bids on nearby houses.

The $W\underline{X}\alpha$ term in (2) allows the structural characteristics of neighboring houses to influence the price of each house. A common saying in real estate is to never own the largest (or the smallest) house on the block: the market will force such a house to sell at a discount, an example of the type of impact captured by $W\underline{X}\alpha$. The $W\underline{X}\alpha$ term also captures the influence that the neighborhood characteristics of nearby houses have on the sale price of each house. For example, many neighboring houses will be in the same school district as the observation in question, but not if the house is near the edge of the school district. The $W\underline{X}\alpha$ term allows a neighboring school district's provision to spill over--possibly through peer group effects--and influence the price of the house in the original school district.

The model in Equation (2) uses maximum likelihood, which may help with identification of the demand curve beyond the traditional method of market segmentation (Epple, 1987). The log-likelihood for the model in Equation (2), concentrated with respect to the parameters β and σ, takes the following form (Anselin, 1988, p. 181; Pace and Barry, 1997a):

$$ln\ L = C + ln\ |I_n - \rho W| - (n/2)ln(e'e) \qquad (3)$$

where

$$e = e_o - \rho e_d \qquad (4)$$

$$e_o = v - Z\beta_o$$

$$e_d = Wv - Z\beta_d$$

$$\beta_o = (Z'Z)^{-1}Z'v$$

$$\beta_d = (Z'Z)^{-1}Z'Wv$$

$$Z = [X\ WXI]$$

and C is a constant term that does not involve the parameters.

The need to compute the log-determinant of the n by n matrix (I_n - ρW) makes it computationally difficult to solve the maximum likelihood problem in Equation (3). Operation counts for computing the determinant grow with the cube of n for dense matrices. However, W is sparse: most of its elements are zeroes. The sparsity of W may be exploited (Pace, 1997; Pace and Barry, 1997a) so that a personal computer can handle the large data set estimations with computational ease. The Cholesky decomposition is used to compute the log-determinant $|I_n - \rho W|$ over a grid of values for ρ restricted to the interval [0,1]. This log determinant uses the Monte Carlo estimator set forth by Barry and Pace (1999), which allows larger problems to be tackled without the memory requirements or sensitivity to orderings associated with the direct sparse matrix approach. For the current model involving 44,255 observations, maximum likelihood estimation required less than one minute on a desktop computer with a Pentium II processor.

5. Spatial Statistics and Omitted Variable Bias

The sparse spatial Durbin procedure has been demonstrated to greatly improve cross-sectional regression estimates that are spatial in nature (Pace, 1998; Pace and Barry, 1997b). Part of the improvement stems from incorporating the influence of omitted variables (Anselin, 1988, p.103; Pace, et al., 1998). Traditional hedonic estimation does not address omitted variable bias. Attempts to circumvent the problem include focusing on narrow geographic areas where many influences are already controlled for, or including vast numbers of explanatory variables to capture every influence which diligent data collection can offer. Still, studies with limited geographic coverage have limited appeal, and structural characteristics may be similar within small areas so that multicollinearity problems are exacerbated. In fact, in most papers it is precisely the variation across markets which enables identification of Rosen's second-stage demand curve from the first-stage hedonics. In addition, no matter how large the number of explanatory variables, regressions may still omit important influences like air quality, landscaping quality, and proximity to parks. Because it incorporates the influence of omitted variables, spatial statistics can improve explanatory power and reduce the parameter estimate bias that generally results from omitting a relevant variable.

Similar to the way a time lag of the dependent variable picks up unobserved autoregressive influences, the spatial lag term ρWv picks up unobserved influences that affect house value (Bolduc et al., 1995; Griffith, 1988, p.82-83). But while a lagged dependent variable in time series regressions relies on observations nearby in time, the spatial lag relies on a linear combination of house values nearby in space. Unmeasured influences help determine the value of neighboring houses and, as explained earlier, the value of neighboring houses is related to the value of our own house. So our own house value is affected by the unmeasured influences of neighboring observations. And the unmeasured influences of neighboring houses are similar to the unmeasured influences for our house because our neighbors are close: the same things that affect our neighbors should affect us, too. So the ρWv term in Equation (2) incorporates the influence of omitted variables on the value of our own house. In fact, with a small number of neighbors used to create W, the ρWv term acts like a highly-localized fixed effects dummy. In a similar manner, the $W\underline{X}\alpha$ term also helps capture the influence of omitted variables whose effects would otherwise be subsumed in the error term and contaminate parameter estimates.

The spatial Durbin model thus captures the influence of air pollution, the presence of shopping centers, interstate highways, lakes, hospitals, and all other omitted variables that vary across space.[2] Dubin (1998) even demonstrates that it is better to allow spatial autocorrelation to capture neighborhood effects than to explicitly include neighborhood variables like income and air pollution levels in house price estimations. In the presence of omitted variable bias, least squares estimates are plagued by a multitude of econometric sins. A detailed proof of how spatial statistics achieves consistent and unbiased parameter estimates, unbiased estimates of the standard errors, and efficient parameter estimates where least squares may not, is available in Griffith (1988, p. 94-107).

6. Choice of Hedonic Variables: House Characteristics and the Measures of Schooling

The house characteristics used are an air conditioning dummy, a fireplace dummy, the number of outbuildings, square feet of lot, age of the house, size of the house, a garage dummy, the number of full and partial bathrooms, and dummy variables for the presence of porches, patios, decks and pools. The squares of lot size, house size, and age are included because these variables may influence a house's value in a nonlinear fashion. Quarter of sale dummies are included to capture any seasonal effects.

Many community variables may affect house price. These variables include racial composition, education levels, income levels, poverty and tax rates, pollution levels, and many more. But as discussed above, it is not necessary to include these variables. Spatial statistics captures the effect of omitting variables whose values differ across space. Racial composition, education levels, income levels, poverty and tax rates, and pollution all vary across space; even if measures of such influences are not explicitly included in the hedonic, their effects will be controlled for by the spatial technique. In fact, in unreported regressions, including such influences explicitly in the hedonics did not appreciably affect the results. The only non-house variables that must be included are the variables from which implicit prices are to be calculated. These variables are public schooling and private schooling.

Public schooling is one of the most important determinants of house price. Recent house price hedonic studies have measured school quality by per-pupil expenditures, proficiency test scores , and value added measures. Brasington and Haurin (2004) reject the use of value added measures. They find proficiency test scores are the most consistently valued measure of school quality, with expenditures a reasonable substitute when proficiency tests are unavailable. In fact, in unreported regressions only three of the six urban areas in the current sample show a significant, positive relationship between the value-added of a school and house prices, making value-added an inferior measure of public school attractiveness. Although expenditures will be tried also, the main school performance variable adopted is the average passage rate on the Ohio 9th-grade proficiency test. The sections are reading, writing, math, and citizenship.

But if a household opts out of public schools, won't that eliminate the relationship between house prices and public schooling? No. The value of public schooling services is capitalized into housing prices whether the household uses public schools, or it uses private schools, or it has no school-aged children. Even if a household doesn't use the public schools, the house could be sold to someone who would. The better are the public schools, the more a potential buyer who uses public schools would be willing to pay.

The same is true for private schools. Even if a household doesn't use private schools, the attractiveness of having good private schools nearby may still be reflected in the price of housing. Previous studies have investigated such a relationship. Seiler (1996) finds that the percentage of students in a public school district who are enrolled in private schools is positively but not significantly related to house prices, while Hayes and Taylor (1996) find a positive, statistically significant relationship. The current study follows Seiler (1996) and Hayes and Taylor (1996) by using private school market share to capture the attractiveness of private schools. Calculating the implicit price from private school market share in a house price hedonic as the price of private schooling has many advantages over using private school tuition. Unlike tuition, the implicit price 1) is a unit price, 2) is based on market prices, 3) uses readily-available Census data, 4) has precedent in the house price hedonic literature, 5) captures all aspects of the attractiveness of private schools, and 6) solves the problem of matching public school districts to private school users. A two-page discussion of the inadequacy of using tuition is available from the author.

7. Data

The primary source of data for the hedonic regressions is a set of arm's length, single-family detached housing purchases that occurred during 1991 in Ohio (Amerestate, 1991). There are 44,255 houses in the 140 school districts in Ohio's six major urban areas, and the mean deflated sale price is $72,809.

ACCRA (1991, 1992) provides the data from which urban area deflators are constructed. The Ohio Department of Education and the U.S. Bureau of the Census (1990) provide the remainder of the explanatory variables. Table 1 shows definitions, data sources, and means of the second-stage demand variables. Table 2 contains means of the variables used in the first-stage house price hedonic regressions.

(see Table 1)

(see Table 2)

8. First Stage Hedonic House Price Estimation Results

The results of the six house price hedonic regressions are shown in Table 3.

(see Table 3)

The hedonic regressions in Table 3 explain a large percentage of the variation in house price; adjusted R-squared ranges from 0.69 in Akron to 0.91 in Dayton. The optimal spatial lag coefficient ρ ranges from 0.23 to 0.49. This suggests moderate spatial autoregressive effects. Likelihood ratio tests reject the null of no spatial dependence at the 1% level of significance for all six hedonic estimations, suggesting that the incorporation of spatial statistics helps prevent omitted variable bias.[3] The parameter estimates of house characteristics conform well to expectations. Outbuildings are generally not related to house price, and the presence of a patio is only measured with precision in three of the six markets, but the other eleven house characteristics generally have the expected relation with house price.

The private and public school variables also conform to expectations. Consistent with Hayes and Taylor (1996) and Seiler (1996), the proportion of high school-aged students attending private school has a positive, statistically significant association with house prices in each of the six regressions.[4] The public school variable, LOG TEST SCORE, is positive and significant in every hedonic. Akron has the highest coefficient estimate: 0.281. In contrast, Cincinnati has the lowest: 0.104. The parameter estimates differ because there are different supply and demand conditions for schooling in each urban area. The availability of housing in school districts of different quality is not uniform, and there is a different supply of public and private schools in each urban area, which contributes to the different parameter estimates. Having variation in the parameter estimates helps achieve identification in the manner Palmquist (1984), Epple (1987), Brown and Rosen (1982), Zabel and Kiel (2000), and Brasington (2003) suggest.

9. Calculation of the Implicit Prices

The results of the hedonic regressions are now used to calculate the implicit prices of public and private schooling. Whereas there were six segmented housing markets for the hedonic regressions, we move to estimating a single demand equation. Assuming a common demand structure in the demand estimation achieves identification in the manner of Brown and Rosen (1982) and others.

The implicit price of public schooling is the partial derivative of house value with respect to the public schooling variable, as explained by Rosen (1974) and used in numerous studies involving non-market goods like schooling. To calculate the implicit prices of public schooling, take the price of each house and multiply it by the coefficient of public schooling in the appropriate hedonic in Table 3. Then divide the product of these two numbers by the relevant proficiency test score. This formula stems directly from the double-log functional form used, although linear, semi-log, and double-log functional forms have all been considered in the literature.[5] The implicit price of private schooling is calculated in much the same way. A series of Davidson and MacKinnon (1981) tests for functional form proved inconclusive, so the log-linear functional form was chosen, following the nearly unanimous precedent of the house price hedonic literature. However, graphs suggested that proficiency test scores should be logged, and the use of LOG TEST SCORE did improve fit over its un-logged counterpart. Because individual house prices enter into the calculation, the implicit prices reflect the fact that homeowners pay different prices for schooling depending on the value of their house (Nechyba, 1999). Following Rosen (1974) and others, the implicit prices of public and private schooling are used in the demand for public schooling estimation.

Finding the price of schooling is but one of several issues in estimating demand; another issue is addressing spillovers.

10. Spillovers in the Demand for Public Schooling

The following theoretical model of the demand for public schooling is adopted from Murdoch, et al.'s (1993) model of recreation expenditures. The primary purpose of the model is to motivate an empirical estimation technique that accounts for spillovers. The theoretical model also helps motivate the choice of explanatory variables in the demand estimation. Let the median voter in school district *j* have the following utility function:

$$U_j = U_j\ (n_j,\ x_j,\ \Omega_j) \tag{5}$$

where n is consumption of a numeraire good, x is consumption of the pure private component of schooling, and Ω is total consumption of the public aspect of public schooling, and U is twice continuously differentiable, strictly quasi-concave and a strictly increasing function of its arguments. An unusual feature of this model is its incorporation of possible external benefits to the production of public education which may spill across jurisdictional boundaries. Formally,

$$\Omega_j = q_j + \omega_j \tag{6}$$

so that total consumption of the public aspect of schooling Ω by members of school district j is determined by own provision of the public component of education (q) and spill-ins from provision in neighboring communities (ω). Each community undertakes an activity g: providing public schooling. This activity jointly produces x and q, with the benefits of x staying strictly within the community but allowing q to spill over into neighboring school districts. The following joint product technology (Sandler, 1977) is assumed:

$$x_j = \theta_j{*}g_j \tag{7}$$

$$q_j = \phi_j{*}g_j \tag{8}$$

so that a certain fraction θ and ϕ of public education provision g goes toward generation of the private component and the pure public component of schooling. Because Equation (8) holds for all school districts,

$$\omega_j = \sum_{i \neq j} (\phi_i \bullet g_i) \tag{9}$$

where ϕ_i is the fraction of activity in school district i that spills in to jurisdiction j as a public good. Spillovers from education are expected to be inversely related to the distance between the school districts in question. One way to account for spillovers is to incorporate spatial dependence in the statistical estimation (Anselin, 1988). From Equations (6), (7) and (8) the median voter's utility function may be rewritten as follows:

$$U_j = U_j\ (n_j,\ \theta_j{*}g_j,\ \phi_j{*}g_j + \omega_j) \tag{10}$$

Utility is maximized by choosing g_j and n_j subject to the following budget constraint:

$$y_j = n_j + \tau_j g_j \tag{11}$$

where y is the median voter's income and τ is the per-unit cost of education faced by the median voter. Utility maximization may proceed in the usual manner.

11. Choice of Explanatory Variables for the Demand Estimation

The dependent variable is the price of public schooling derived from the first-stage hedonics, and so the quantity of public schooling, LOG TEST SCORE, must be included as an explanatory variable.[6] But a demand curve also includes the price of related goods. Private schooling may be a substitute for public schooling, so the implicit price of private schooling is included in the demand estimation. Both LOG TEST SCORE and PRICE OF PRIVATE EDUCATION are treated as endogenous variables.[7]

Demographic variables hypothesized to influence demand are income levels and the proportion of households who own their residence rather than rent. These variables either appear directly in the theoretical model or influence the choice of g (public schooling provision) relative to n (the numeraire good).

Rubinfeld, et al. (1987) inspire the inclusion of two sorting variables to help mitigate Tiebout bias: the percentage of residents who have lived in a community for less than six years and a central city dummy.[8]

12. A Demand Estimation Technique that Addresses Spillovers and Omitted Variable Bias

The theoretical model of demand suggests spillovers. If the schooling provision of one community influences provision in a neighboring community, spatial dependence exists. Introductory textbooks commonly use public schooling as an example of a good that exhibits positive externalities, so it may be worthwhile to use spatial statistics, which can capture the influence of spillovers (Anselin, 1988). Papers that use spatial statistics to capture externalities in public good provision include Murdoch, et al. (1993), Case, et al. (1993), Brueckner (1998), and Brasington and Hite (2004). The same spatial Durbin model used in the house price hedonics is used to estimate the following inverse demand curve:

$$P = \rho WP + X\beta + W\underline{X}\alpha + \varepsilon,\ \varepsilon \sim N(0, \sigma^2 I_n) \qquad (12)$$

The spatial autoregressive term ρWP in Equation (12) suggests that the price of public schooling is most similar for nearby houses and school districts, and less similar for houses and school districts farther away. The ρWP term addresses omitted variable bias in the same way as described in Section 5 for the house price hedonics. But in the demand curve the $W\underline{X}\alpha$ term may play the more important role.

The price of public schooling in a particular community may be more than a function of its own demographic factors and quantity. It may also be related to demographic influences in neighboring communities. For instance, schooling in district A may be related to A's income levels, but it may also be related to income levels in neighboring district B. After all, children on the border of two school districts may play with each other, and parents on the border of two school districts may interact and influence each other's desire for public schooling. Churches, women's clubs, and Rotary clubs draw members from different nearby school districts; members of these groups interact and may directly or indirectly influence attitudes about public schooling (Case and Katz, 1991). The $W\underline{X}\alpha$ term allows for these types of influences.

The $W\underline{X}\alpha$ term also allows for spillovers in the provision of schooling. The price of schooling is related to its own quantity of schooling, but if there are spillovers to the provision of schooling, the amount of schooling provided by neighboring school districts affects our own provision. We may partially free ride off the spillovers from neighboring districts, as Equation (6) from the theoretical model shows. So a neighbor's value of LOG TEST SCORE may affect our own value of LOG TEST SCORE, which in turn affects our PRICE OF PUBLIC EDUCATION. Econometrically, from Equation (12), $W\underline{X}$ may be related to P when X is the quantity of public schooling. Failure to include the $W\underline{X}\alpha$ term may open the regression to omitted variable bias.

So to address the potential influence of both spillovers and omitted variable bias, demand is now estimated using the spatial Durbin model.

13. Second Stage: Demand for Education Results

The results of the demand for public education regression are shown in Table 4.

(see Table 4)

Second-stage hedonic demand estimations have traditionally used two-stage least squares (2SLS) and limited information maximum likelihood (LIML). Demand is first estimated using these traditional approaches so the impact of using spatial statistics in the second stage may be assessed. The 2SLS column of Table 4 shows a price elasticity of demand for public schooling of -0.80.[9] Most demand studies report a more inelastic estimate ranging from -0.20 to -0.40 (Reiter and Weichenrieder, 1997), and a non-spatial demand study using a similar Ohio data set reports an elasticity of -0.11 (Brasington, 2002). But the price variables in the current study are calculated from house price hedonics that address spatial dependence. The 2SLS results suggest that ignoring spatial dependence in the first stage biases the second-stage price elasticities upward. In fact, when spatial statistics is also used in the second stage, the price elasticity of demand estimate will be shown to rise even more. The 2SLS column shows a cross-price elasticity of 0.83, suggesting that public and private schooling are substitutes. But it also suggests a negative income elasticity of demand, contrary to theory. No strong interpretation is given to CENTRAL CITY and PERCENT NEWCOMERS; they are merely included to address Tiebout bias. A higher home ownership rate is positively related to public schooling demand, supporting the theory that homeowners have more to gain than renters from the capitalization of public schooling into house prices.

The LIML column of results in Table 4 provides similar estimates to those of 2SLS: own price elasticity changes from -0.80 to -0.78, the cross price elasticity changes from 0.83 to 0.91, and the income elasticity remains negative. Adjusted R-squared drops from 0.40 to 0.38.

The Baseline column of Table 4 uses the spatial Durbin model to estimate the demand for public schooling. First, note that adjusted R-squared rises from 0.40 to 0.57. Second, the spatial autoregressive parameter has a 0.32 estimate, which suggests that the price of public schooling is somewhat geographically related. Failure to include the ρWP term has subjected the 2SLS and LIML models to omitted variable bias. The non-spatial price elasticity of demand is estimated at -0.11; when spatial dependence is addressed in the first stage only, the price elasticity becomes -0.80; and now, when spatial dependence is addressed in both the first stage and the second stage demand estimation, price elasticity becomes -1.32. At each stage omitted variables seem to bias the price elasticity estimates upward. By addressing omitted variable bias, demand is shown to be more price elastic than previously thought.

The Baseline estimate of the cross-price elasticity between public and private schooling is 0.66. Allen and Shen (1999) find the cross-price elasticity between private college enrollments and public university tuition is –0.02. One may debate how comparable the elasticities are between the different levels of schooling, but the literature provides little comparison for the 0.66 elasticity.

The 0.66 cross-price elasticity must be carefully interpreted, both in what it says and what it cannot say. It is posivite, which says private schooling is a substitute for public schooling. The literal interpretation is that, at the mean, a one percent decrease in the price of private schooling would be associated with a 0.66

percent fall in the demand for public schooling. But if average private school tuition were $3500, a one percent fall in the price of private schooling would amount to less than a $100 voucher. All partial voucher proposals are more generous than this. Suppose President Bush's $1500 voucher bill had passed in 2001. Suppose average private school tuition were $3500 and parents could supplement the voucher with additional money. It is also important to consider public school taxes. Average expenditures per public school student is currently $7000. But because not every adult (or corporation) has children, and the amount everyone pays toward public schools varies by person, it is difficult to estimate how much the typical person pays for public schools. For a rough approximation, through property and income and sales taxes, suppose the typical parent pays $3000 for public schools, which gets paid whether the parent uses public school or not. In these circumstances a $1500 voucher would reduce the price of attending private schools from ($3000 + $3500 =) $6500 to ($3000 + $3500 - $1500) = $5000, reducing the price of private schooling by about 23%. The 0.66 elasticity implies that the $1500 voucher would reduce the demand for public schooling by (0.23 * 0.66 =) 15.2%.

The mechanisms behind the 15.2% reduced demand could take many forms, but the end result is a 15.2% lower amount of public schooling demanded. Exactly what combination of student transfers from public schools, reduced public school expenditures, and cream-skimming cause the 15.2% decline cannot be specified, but flight from public schools, reduced expenditures, and cream-skimming are inter-related. As the brightest students flee the public schools (Epple and Romano, 1998), the lower peer effects depress public school outcomes. As these and other students take their voucher money to private schools, public school budgets are cut, perhaps leading to larger class sizes and lower schooling outcomes. And as the constituent base of the public schools erodes, tax levies are more difficult to pass, also reducing school budgets. The numerical example of a $1500 voucher possibly leading to a 15.2% drop gives the impression that, at least in the short run, a voucher system would not cause people to demand much less public schooling, and something less than a mass exodus of parents would switch from public to private schools. But there are other effects of vouchers, which are touched upon in the conclusion.

The Baseline results of Table 4 also show that income levels are not significantly related to the quantity of public schooling demanded, holding the other controls constant. The adoption of the spatial Durbin technique has eliminated the downward bias that caused the negative income elasticities in the 2SLS and LIML models. And while home ownership is still positively related to public schooling demand, the spatial Durbin model decreases the parameter estimate from 0.72 to 0.16.

Finally, the spatial Durbin shows how the explanatory variables of our neighbors are related to our own demand for public schooling. Four of these six SPATIAL LAG variables are statistically significant, suggesting that their inclusion helps avoid omitted variable bias. The most interesting of these is SPATIAL LAG LOG TEST SCORE. Its 0.15 parameter estimate suggests that there are spillovers to the provision of public schooling, making the current study one of the few to attempt to quantify these spillovers. The literal interpretation is that a one percentage point increase in our neighbors' quantity of public schooling is associated with a $4.50 rise in the price of a unit of our public schooling.[10] The mechanism of the price rise is consistent with spillovers: when our neighbors provide more public schooling, we free ride by cutting back on our quantity, and decreasing our quantity along a downward-sloping demand curve is associated with a rise in price.

The spatial Durbin technique is less common than the spatial autoregressive (SAR) model, so an exploration of SAR may be warranted. The SAR model is the same as Equation (12) with α set to zero:

$$P = \rho WP + X\beta + \varepsilon,\ \varepsilon \sim N(0, \sigma^2 I_n) \qquad (13)$$

The results are found in the SAR column of Table 4. Most results do not change markedly. The spatial autoregressive parameter's estimate falls from 0.32 to 0.31, price elasticity of demand grows from -1.32 to -1.36, the cross-price elasticity goes from 0.66 to 0.86, the parameter estimate of OWNER OCCUPIED goes from 0.16 to 0.15. On the other hand, explanatory power falls from 0.57 to 0.42, confirming that the spatial lag terms *WX* are important. In fact, the spatial lag terms seem to have been responsible for eliminating the downward bias on the income variable. The 2SLS results show a negative income parameter estimate of -1.49; the SAR model captured some of the omitted variable bias, bringing the income parameter estimate to -0.38; but it took the spatial lag terms to bring the parameter estimate to a more theoretically consistent (if statistically insignificant) estimate of 0.12.

14. Second Stage Continued: More Neighbors, Small v Large Houses, and Expenditures

The preceding analysis uses a small number of neighbors to form the spatial weight matrix. The advantage of this approach is to capture highly localized influences not explicitly included in the demand equation. The disadvantage is that it fails to pick up much influence from neighboring school districts: a vast majority of the time, the five houses closest to our own will all be in the same school district. The theoretical model of Section 10 emphasizes the possibility of spillovers between school districts. To allow the spatial Durbin model to capture more potential spillovers, the spatial weight matrix is now based on the nearest 200 neighbors. The typical school district contains 316 houses, so using 200 neighbors retains a component of neighbors from within the same school district while adding a component from nearby school districts. Computational power becomes more of an issue, but still, thanks to Pace (1997; and Barry, 1997a) once the weight matrix has been created, the 200-neighbor, 44,255 observation spatial Durbin model takes just over half an hour to run.

(see Table 5)

The first column of results in Table 5 replicates the Baseline result of Table 4 using 200 neighbors instead of five. There is no change in statistical significance of any regressor, although the increased proportion of neighbors from other school districts has caused three of the spatial lags to change their statistical significance. The price elasticity of demand has gone from -1.32 to -2.02, and the cross-price elasticity has gone from 0.66 with five neighbors to 0.93 with 200 neighbors. The second column of results in Table 5 compares the spatial autoregressive model to the spatial Durbin model. As in the five-neighbor case, the spatial autoregressive model for the 200-neighbor case reveals a negative, significant parameter estimate for income, confirming the choice of the spatial Durbin model. On the other hand, the price elasticity (-1.71) and cross-price elasticity (0.87) in the spatial autoregressive model are similar to those in the spatial Durbin model.

Houses with a large number of bedrooms may be more likely to contain children than houses with few bedrooms. If having more bedrooms makes a house more marketable to families that use public schools, the implicit price of school quality may be stronger for houses with more bedrooms. So houses with many bedrooms may have different price and cross-price elasticities of demand for public schooling than houses with few bedrooms. The analysis is repeated splitting the sample into houses with three or fewer bedrooms, and houses with four or more bedrooms. The results of the baseline demand curve regression are shown in the Small House Sample and Large House Sample columns of Table 5. The parameter estimates of income and public schooling are similar for the large and small house samples, but the cross-price elasticity differs: it is 0.52 for small houses and 0.84 for large houses. The data suggest that families are more responsive to changes in the price of private schooling than households without children. The results appear to not to stem from a high correlation between the number of bedrooms and income.[11]

Finally, although proficiency tests have largely replaced per-pupil expenditures as school quality measures in house price hedonics, and Brasington and Haurin (2004) find proficiency tests superior, expenditures still represent valid measures of school quality and are occasionally adopted in the recent literature. The results of the spatial Durbin demand model when expenditures are used instead of proficiency tests are found in the final column of Table 5. The use of expenditures yields a much less price elastic demand for public schooling (-0.014), but the cross-price elasticity of 0.77 closely approximates the 0.66 cross-price elasticity achieved by the model that uses proficiency test scores.

15. Flight to Private Schools: From Richer or Poorer?

Proponents of vouchers argue that vouchers would benefit students in low-income districts most; opponents say that the low-income school districts are the most likely to suffer from funding cuts as students flee to private schools. The cross-price elasticity for the Baseline results can be calculated for poor and rich homeowners. Assuming income levels and house prices are correlated, evaluating the cross-price elasticity for the 25th and 75th percentiles of house price can show how differently the rich and poor view public and private schools to be substitutes.[12]

Poor homeowners have a cross-price elasticity of 0.43; it is 0.82 for the rich.[13] Contrary to popular speculation, the results of the current study suggest that private schools are stronger substitutes for the rich than for the poor: flight from public schools is likely to be more pronounced in high-income school districts. The 0.43 elasticity suggests some poor students will redeem school vouchers, but it is the rich schools that may face more financial difficulties. By the same token, vouchers are often presumed to help poor students have better educational opportunities at private schools, but it is the rich who would most likely redeem school vouchers. This is exactly what happened when Britain introduced a voucher system (Willms and Echols, 1992). And because private schools are stronger substitutes for high-income public schools, the high-income public schools are the ones who would face the strongest incentives to improve their performance, leaving low-income public schools less subject to competitive pressures.

16. Conclusions

Will school choice provide competition for public schools? The answer depends largely on the degree to which public and private schooling are substitutes. The current study estimates the demand for public schooling and finds the cross-price elasticity between public schooling and the price of private schooling is between 0.66 and 0.93. Public and private schooling seem to be moderate substitutes. That they are moderate substitutes suggests that a voucher system or a tuition tax credit that makes private schools more affordable will not cause a mass exodus from public schools, at least not immediately. The results of the current study must be cast in light of previous research on the demand for private schooling and the experiences of other nations with voucher systems.

There seem to be no other demand for K-12 public schooling studies that calculate the cross-price elasticity of private schooling. However, there are studies that estimate the demand for private schooling. Some studies find the demand for private schooling is sensitive to changes in private school tuition (Sandy, 1992; West and Palsson, 1988), some find the demand for private schooling is not sensitive to changes in private school tuition (Buddin, et al., 1998; Chiswick and Koutromanes, 1996), and others find mixed results depending on whether primary or secondary schools are examined (Sandy, 1992; Hoenack, 1992). The evidence is mixed on whether private school usage is responsive to changes in tuition, providing no clear evidence whether parents would switch from public to private schools if a voucher system or tuition tax credits made private schools cheaper.[14] Another way to see whether parents would switch to private schools is to see what happened in other countries that have enacted voucher plans.

The United States is not the only country to experiment with school vouchers. Bangladesh, Belize, Chile, Colombia, Czech Republic, Guatemala, Ivory Coast, Lesotho, Poland, Sweden, and the United Kingdom have voucher systems where the vouchers may be used toward the public or private school of the parents' choice. British Columbia, Quebec, Manitoba, Saskatchewan, Alberta, Japan, the Netherlands, Belgium, France, and New Zealand also have state support for private schools.[15] While Sandy (1992) finds Americans with low socio-economic status support vouchers the most, Willms and Echols (1992) find that Scottish parents with high socio-economic status were the ones who took advantage of Britain's school voucher system. Did the introduction of vouchers in other nations cause a mass exodus from the public schools? The proportion of students attending private schools more than doubled eight years after Chile adopted private school vouchers. On the other hand, just 17% of voucher recipients in Puerto Rico switched from public to private schools; most Puerto Rican voucher recipients transferred from one public school to another (West, 1997). Belgium now has one of the highest private school attendance rates in the world, but New Zealand's private school attendance rate has hardly changed since the government began subsidizing private schools in 1974 (Toma, 1996).

If the experiences of other nations were similar, perhaps conclusions could be drawn about the likely effect of school vouchers and private school tuition tax credits in the United States. But they are not. The response of each nation may depend on how easily each nation's residents believe private schools may be substituted for public schools. The current study's cross-price elasticity of 0.66 suggests that public and private schooling are not powerful substitutes in the United States, but neither are they weak substitutes. It is dangerous to interpret regression results beyond small, marginal changes in the explanatory variables. But given the lack of guidance from previous literature, and the importance of the public policy question, any information we can get on the possible effects of a generous voucher program may be worth the risk of stretching the 0.66 cross-price elasticity beyond its limits. This said, a voucher program as generous as Colombia's that covers half the cost of private schooling may decrease the demand for public schooling by 33%.

Some opponents of school vouchers in the United States argue that vouchers would cause Americans to flock to private schools en masse, leaving the poor in gutted, underfunded, and decaying public schools (e.g., Krashinsky, 1986; Ansell, 2003). The cross-price elasticities of the current study do not support such a scenario. If the 33% number means that 33% of public school students would switch to private schools—which is one of many possible interpretations of the 33% number—public schools would lose their dominant market share. However, most private school systems are already near full capacity, so not all 33% of public school users could switch to private schools. Faced with excess demand, private schools may raise tuition rates to exclude almost all additional customers, stemming the 'mass exodus' from public schools until more private schools can be built.

The results further suggest that private schools are stronger substitutes for high-income public school districts than for low-income public school districts, so a voucher system seems to pose a greater long-term financial threat to high-income public school districts. High-income public school districts are more likely to face a mass exodus and loss of funding, and therefore to face a stronger incentive to improve performance. By the same token, tuition tax credits and school voucher programs that limit participation to low-income households will likely experience lackluster participation, which would both lower the cost of providing these programs and help shield failing public schools from competition.

The results of the current study must also account for the lack of variety seen in private schools today. Existing voucher experiments in the United States are small programs with strictly limited participation (Merrifield, 2002). Such experiments provide little clue about how many students would switch if there were widespread voucher programs. The current study is based on the premise that public schools already face a certain degree of competition from existing private schools. By studying the elasticity of substitution between them, it can provide some clue about how many students would take advantage of private school vouchers and tuition tax credits. But the 0.66 elasticity of substitution generated by today's data may underestimate the long-run response to vouchers. Today, parents must choose between "free" public schools and a limited set of private school alternatives. But as Merrifield (2002) notes, a voucher system may spawn a supply-side response that yields a vast array of private schools with innovative, attractive educational menus. As the alternatives become more attractive, a larger proportion of parents will probably redeem their vouchers at private schools, so the cross-price elasticity of the current study may best be viewed a lower-bound, short-run estimate of the degree to which public schools might lose support to private schools under a voucher program.

In short, given the 0.66 cross-price elasticity, capacity constraints, and the lack of variety of types of private schools, the current study suggests that if vouchers or tuition tax credits were enacted, public schools would have some time to compete to retain their students. Even though the move from public to private schools would at first resemble a steady stream rather than a torrent, the public schools may still respond to the increased competition. The mere introduction of school choice may scare public schools into greater efficiency and effectiveness, encouraging parents to continue to support the public schools with their tax levies, school voucher money, and talented children; and limiting the market share of private schools to something close to the 12% it currently commands.

The study supports the following additional findings. Own price elasticity of demand for the model that uses spatial statistics in both stages is –1.32; when spatial statistics is used in the second stage only it drops to -0.80; and a non-spatial model shows a -0.11 elasticity. The spatial Durbin model seems to reveal a more price elastic demand curve than studies that fail to address omitted variable bias. The spatial Durbin technique used in the demand estimation finds evidence of small spillovers across public school districts. The spatial Durbin model yields a somewhat higher explanatory power and more theoretically consistent parameter estimates than 2SLS, LIML, or even a spatial autoregressive model.

Table 1
Variable Definitions, Sources and Means

LOG HOUSE PRICE	Natural log of house transaction price in U.S. dollars for 1991 sales, deflated by urban area cost of living index (1,4)	For means of housing variables see Table 2
AIR CONDITIONING	Air conditioning dummy (1)	
FIREPLACE	Fireplace dummy (1)	
OUTBUILDINGS	Number of detached structures on lot (1)	
LOT SIZE	Size of lot in tens of thousands of square feet (1)	
AGE	Age of house in hundreds of years (1)	
HOUSE SIZE	Thousands of square feet of house size (1)	
GARAGE	Garage dummy (1)	
FULL BATHROOMS	Number of full bathrooms (1)	
PART BATHROOMS	Number of partial bathrooms (1)	
PORCH	Porch dummy (1)	
PATIO	Patio dummy (1)	
DECK	Deck dummy (1)	
POOL	Pool dummy (1)	
%PRIVATE	Proportion of persons in census block group aged 14-17 who attend private school (2)	0.23 (0.19)
LOG TEST SCORE	Natural log of the percentage of ninth-grade students passing the Ohio 9th grade proficiency test in 1990. Average passage rate of math, reading, writing, and citizenship sections; unlogged mean = 33.5 (3)	3.35 (0.60)
LOG EXPENDITURES	Natural log of total school district expenditures per pupil in U.S. dollars, deflated by urban area cost of living index (3,4)	8.50 (0.14)
PRICE OF PUBLIC EDUCATION	Tax-adjusted implicit price of public education in hundreds of dollars, derived from hedonic regressions as partial derivative of house price with respect to test scores or expenditures	4.57 (3.26)
PRICE OF PRIVATE EDUCATION	Tax-adjusted implicit price of the attractiveness of private schooling in thousands of dollars, derived from hedonic regressions as partial derivative of house price with respect to private school attendance rate	15.25 (7.37)
MEDIAN INCOME	Median of the average income of persons living in each census block group comprising each school district, in hundreds of thousands of urban area-deflated dollars (2,4)	0.41 (0.19)
CENTRAL CITY	Dummy variable for whether the school district is a central city school district	0.35 (0.48)
PERCENT NEWCOMERS	Proportion of persons living in each census block group comprising each school district who have lived in their current location less than six years (2)	0.45 (0.09)
OWNER OCCUPIED	Proportion of persons living in each census block group comprising each school district who own their residence rather than rent (2)	0.74 (0.20)
SPATIAL LAG ----	Spatial lag of the explanatory variable named, the $W\underline{X}$ terms from Equation (2) in text	- -

Sources: (1) Amerestate (1991) housing transaction data set; (2) U.S. Bureau of the Census; (3) Ohio Department of Education, Division of Education Management Information Services; (4) ACCRA (1991, 1992). Number of observations = 44,255 houses in 140 urban school districts from the six largest urban areas in Ohio: Akron, Cincinnati, Cleveland, Columbus, Dayton, Toledo.

Table 2
Hedonic Means (and Standard Deviation) by Urban Area

Variable	Akron	Cincinnati	Cleveland	Columbus	Dayton	Toledo
LOG HOUSE PRICE	10.87	11.12	11.01	11.05	11.10	10.94
	(0.67)	(0.56)	(0.60)	(0.59)	(0.58)	(0.66)
HOUSE PRICE	65,056	78,580	71,053	73,708	77,135	69,102
	(43,854)	(44,552)	(40,909)	(41,556)	(41,557)	(44,194)
LOG TEST SCORE	3.44	3.45	3.30	3.34	3.32	3.27
	(0.41)	(0.42)	(0.69)	(0.59)	(0.74)	(0.48)
%PRIVATE	0.16	0.27	0.27	0.17	0.19	0.28
	(0.15)	(0.20)	(0.21)	(0.17)	(0.17)	(0.18)
LOG EXPENDITURES	8.43	8.45	8.51	8.55	8.50	8.50
	(0.08)	(0.17)	(0.17)	(0.12)	(0.11)	(0.06)
AIR CONDITIONING	0.22	0.52	0.19	0.50	0.47	0.35
	(0.42)	(0.50)	(0.39)	(0.50)	(0.50)	(0.48)
FIREPLACE	0.35	0.42	0.33	0.45	0.45	0.35
	(0.48)	(0.49)	(0.47)	(0.50)	(0.50)	(0.48)
OUTBUILDINGS	0.050	0.052	0.012	0.019	0.040	0.037
	(0.23)	(0.24)	(0.11)	(0.14)	(0.20)	(0.21)
LOT SIZE	1.16	1.18	1.06	0.89	1.08	0.91
	(1.01)	(0.97)	(1.04)	(0.61)	(0.71)	(0.68)
LOT SIZE SQUARED	2.37	2.33	2.20	1.17	1.67	1.28
	(5.58)	(5.73)	(6.90)	(3.51)	(3.42)	(3.37)
AGE	0.47	0.43	0.46	0.36	0.39	0.42
	(0.24)	(0.26)	(0.24)	(0.23)	(0.23)	(0.26)
AGE SQUARED	0.28	0.26	0.27	0.18	0.20	0.25
	(0.25)	(0.28)	(0.27)	(0.22)	(0.24)	(0.27)
HOUSE SIZE	1.38	1.48	1.45	1.45	1.48	1.45
	(0.48)	(0.52)	(0.49)	(0.47)	(0.54)	(0.54)
HOUSE SIZE	2.15	2.47	2.35	2.32	2.47	2.41
SQUARED	(1.62)	(1.83)	(1.75)	(1.62)	(1.90)	(1.92)
GARAGE	0.85	0.82	0.92	0.75	0.87	0.89
	(0.36)	(0.39)	(0.27)	(0.43)	(0.33)	(0.31)
FULL BATHROOMS	1.24	1.39	1.21	1.33	1.42	1.23
	(0.46)	(0.56)	(0.44)	(0.49)	(0.53)	(0.45)
PART BATHROOMS	0.33	0.31	0.35	0.40	0.29	0.39
	(0.50)	(0.47)	(0.50)	(0.50)	(0.47)	(0.52)
PORCH	0.67	0.64	0.57	0.57	0.57	0.65
	(0.47)	(0.48)	(0.49)	(0.50)	(0.50)	(0.48)
PATIO	0.008	0.26	0.052	0.32	0.51	0.19
	(0.09)	(0.44)	(0.22)	(0.47)	(0.50)	(0.39)
DECK	0.072	0.15	0.11	0.13	0.085	0.088
	(0.26)	(0.36)	(0.31)	(0.33)	(0.28)	(0.28)
POOL	0.006	0.028	0.010	0.006	0.013	0.024
	(0.074)	(0.164)	(0.099)	(0.080)	(0.114)	(0.153)
Observations	5018	7148	13723	7680	6779	3907

Table 3
Spatial Hedonic Results with Test Scores

Variable	Akron	Cincinnati	Cleveland	Columbus	Dayton	Toledo
%PRIVATE	0.143** (48.2)	0.221** (68.6)	0.241** (237.2)	0.280** (183.2)	0.091** (61.2)	0.148** (47.0)
LOG TEST SCORE	0.281** (131.8)	0.104** (22.0)	0.244** (774.8)	0.131** (215.6)	0.116** (175.8)	0.149** (72.2)
AIR CONDITIONING	0.090** (44.0)	0.083** (114.8)	0.029** (13.4)	0.074** (66.4)	0.069** (96.6)	0.058** (26.8)
FIREPLACE	0.095** (72.6)	0.086** (108.0)	0.069** (123.8)	0.069** (72.2)	0.053** (46.8)	0.055** (26.0)
OUTBUILDINGS	0.029 (2.0)	0.026 (3.8)	0.000 (0.0)	0.031 (2.6)	0.041** (7.2)	-0.007 (0.4)
LOT SIZE	0.006** (21.2)	0.009** (69.0)	0.007** (58.6)	0.008** (38.4)	0.010** (42.2)	0.007** (27.0)
LOT SIZE SQUARED	0.000** (10.2)	0.000** (30.2)	0.000** (28.2)	0.000** (20.6)	0.000** (18.6)	0.000* (5.6)
AGE	-0.010** (108.6)	-0.005** (108.2)	-0.006** (1147.4)	-0.006** (39.0)	-0.008** (105.8)	-0.003** (41.8)
AGE SQUARED	0.000** (24.2)	0.000** (89.8)	0.000** (182.8)	0.000** (35.8)	0.000** (45.8)	0.000** (94.4)
HOUSE SIZE	0.265** (29.8)	0.341** (96.2)	0.354** (142.8)	0.361** (73.6)	0.336** (94.6)	0.540** (146.4)
HOUSE SIZE SQUARED	-0.008 (2.4)	-0.009 (1.0)	-0.025** (15.0)	-0.021** (10.6)	-0.024** (9.2)	-0.062** (33.0)
GARAGE	0.184** (172.2)	0.103** (127.4)	0.155** (235.4)	0.054** (35.8)	0.146** (260.2)	0.205** (178.4)
FULL BATHROOMS	0.037** (8.0)	0.064** (54.6)	0.044** (27.2)	0.040** (14.4)	0.060** (44.8)	0.083** (36.6)
PART BATHROOMS	0.079** (56.0)	-0.001** (11.6)	0.072** (113.6)	0.037** (15.6)	0.057** (49.2)	0.064** (31.4)
PORCH	-0.008* (4.8)	0.019** (15.0)	0.014* (5.8)	-0.001** (11.6)	0.017* (5.6)	0.023** (8.4)
PATIO	0.040* (5.6)	0.012 (2.6)	0.005 (0.0)	0.020** (13.4)	0.031** (22.0)	0.011 (1.4)
DECK	0.039 (3.8)	0.064** (41.8)	0.041** (30.0)	0.033** (11.4)	0.081** (46.0)	0.057** (13.6)
POOL	0.015 (0.2)	0.045** (9.4)	0.025 (1.2)	0.092* (4.6)	0.064** (7.2)	0.067** (7.0)
CONSTANT	6.386** (807.0)	4.605** (1443.6)	6.911** (204.0)	5.667** (3399.8)	4.817** (1848.0)	6.044** (534.8)
Observations	5018	7148	13723	7680	6779	3907
Log likelihood	-16,071.2	-22,740.4	-49,506.0	-25,451.5	-20,661.0	-11,213.3
Autoregressive lag, ρ	0.28	0.49	0.23	0.37	0.46	0.30
Adjusted R-Squared	0.69	0.87	0.71	0.77	0.91	0.79

Quarter of sale dummies (omitting first quarter) are included in regression to capture seasonal effects but results are suppressed in output. Estimated values of spatial lag estimates $\alpha W\underline{X}$ are also included but suppressed in output. Dependent variable is LOG HOUSE PRICE. Parameter estimates shown with likelihood ratio statistics in parentheses below. **=statistically significant at 0.05, *=statistically significant at 0.10.

Table 4
Demand Estimation Results

Variable Name	2SLS	LIML	*Baseline*	SAR
LOG TEST SCORE	-5.72** (50.6)	-5.87** (48.5)	*-3.47*** *(24.0)*	-3.37** (45.8)
PRICE OF PRIVATE EDUCATION	0.31** (29.0)	0.35** (27.9)	*0.15*** *(6.2)*	0.19** (23.4)
MEDIAN INCOME	-1.49** (6.6)	-2.40** (9.6)	*0.12* *(0.3)*	-0.38* (2.1)
CENTRAL CITY	-1.21** (26.1)	-1.26** (25.6)	*0.21*** *(2.4)*	-0.33** (9.7)
PERCENT NEWCOMERS	-10.94** (41.3)	-11.31** (40.3)	*-5.21*** *(16.9)*	-7.74** (60.2)
OWNER OCCUPIED	0.72** (6.4)	0.71** (6.0)	*0.16** *(1.7)*	0.15* (1.9)
INTERCEPT	24.71** (143.2)	25.30** (136.9)	*15.24*** *(116.5)*	15.44** (4279.5)
SPATIAL LAG LOG TEST SCORE	- -	- -	*0.15** *(1.7)*	- -
SPATIAL LAG PRICE OF PRIVATE EDUCATION	- -	- -	*0.04** *(1.7)*	- -
SPATIAL LAG MEDIAN INCOME	- -	- -	*-0.35* *(0.8)*	- -
SPATIAL LAG CENTRAL CITY	- -	- -	*-0.59*** *(7.2)*	- -
SPATIAL LAG PERCENT NEWCOMERS	- -	- -	*-2.60*** *(6.2)*	- -
SPATIAL LAG OWNER OCCUPIED	- -	- -	*0.00* *(0.0)*	- -
Log likelihood	-	-	-297,786	-83,411
Autoregressive lag, ρ	-	-	0.32	0.31
Adjusted R-Squared	0.40	0.38	0.57	0.42

Dependent variable is PRICE OF PUBLIC EDUCATION. **=significant at 0.05, *=significant at 0.10. Parameter estimates are shown. Absolute value of asymptotic t-statistic shown in parentheses below. Number of observations = 44,255. Endogenous variables: LOG TEST SCORE, PRICE OF PRIVATE EDUCATION. 2SLS = two-stage least squares model. LIML = limited information maximum likelihood model. Baseline = spatial Durbin model. SAR = spatial autoregressive model.

Table 5
Additional Demand Estimation Results

Variable Name	200 Neighbor Baseline	200 Neighbor SAR	Small House Sample	Large House Sample	Expenditures Baseline
LOG EXPENDITURES	-	-	-	-	-0.083**
	-	-	-	-	(15.7)
LOG TEST SCORE	-2.26**	-2.67**	-5.40**	-5.71**	-
	(21.6)	(73.1)	(30.6)	(16.8)	-
PRICE OF PRIVATE EDUCATION	0.14**	0.15**	0.20**	0.25**	0.0049**
	(6.1)	(23.2)	(9.1)	(5.7)	(11.0)
MEDIAN INCOME	-0.53	-0.40**	-0.53	-0.082	0.036**
	(1.2)	(2.4)	(1.4)	(0.2)	(5.8)
CENTRAL CITY	0.89**	-0.25**	-0.91**	-0.69**	0.0043**
	(12.1)	(8.0)	(13.3)	(5.0)	(3.7)
PERCENT NEWCOMERS	-3.62**	-6.14**	-5.63**	-6.05**	-0.14**
	(12.5)	(47.3)	(10.5)	(5.8)	(10.8)
OWNER OCCUPIED	0.82**	0.10	0.12	-0.69**	-0.019**
	(5.8)	(1.2)	(1.2)	(2.9)	(9.0)
INTERCEPT	10.60**	12.28**	-0.076	-1.07*	0.048**
	(131.3)	(137.0)	(0.6)	(2.2)	(10.0)
SPATIAL LAG LOG TEST SCORE	-0.01	-	5.23**	6.01**	0.076**
	(0.3)	-	(31.5)	(20.0)	(11.5)
SPATIAL LAG PRICE OF PRIVATE EDUCATION	-0.009	-	-0.21**	-0.28**	-0.0054**
	(0.4)	-	(9.7)	(5.2)	(25.3)
SPATIAL LAG MEDIAN INCOME	0.26	-	1.69**	0.42	-0.0033
	(0.6)	-	(5.8)	(0.5)	(0.1)
SPATIAL LAG CENTRAL CITY	-1.2**	-	0.79**	0.76**	-0.0058
	(17.5)	-	(19.3)	(3.7)	(1.4)
SPATIAL LAG PERCENT NEWCOMERS	-3.62**	-	5.85**	5.53**	0.15**
	(12.5)	-	(11.3)	(4.1)	(11.1)
SPATIAL LAG OWNER OCCUPIED	-0.87**	-	0.41*	1.85*	0.023**
	(5.3)	-	(1.8)	(2.2)	(10.0)
Log likelihood	-77,752	-79,326	-57,408	-18,315	-92,163
Autoregressive lag, ρ	0.56	0.47	0.98	0.96	0.96
Adjusted R-Squared	0.41	0.40	0.56	0.41	0.24
Observations	44,255	44,255	34,779	9476	44,255

Dependent variable is PRICE OF PUBLIC EDUCATION. **=significant at 0.05, *=significant at 0.10. Parameter estimates are shown. Absolute value of asymptotic t-statistic shown in parentheses below. Endogenous variables: LOG TEST SCORE, PRICE OF PRIVATE EDUCATION. Baseline = spatial Durbin model. SAR = spatial autoregressive model. Small House Sample is houses with three or fewer bedrooms; Large House Sample is houses with four or more bedrooms. Expenditures Baseline is spatial Durbin model using per pupil expenditures as school quality measure instead of proficiency test scores.

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[1] Some studies, like Gemello and Osman (1983), say private school vouchers would raise tax burdens; other studies, like Seldon (1986), say government savings could occur if the value of the voucher were less than the full cost of public schooling.

[2] One influence that does not likely vary over space and is therefore still a problem is people's intrinsic desire for private schooling. For example, if people choose to live in an ethnic community and attend a private, ethnic school, it is difficult to disentangle the intrinsic desire for private schooling from the attractiveness of the private school. The intrinsic desire for private schooling may be part of the error term and correlated with the private school attendance rate, but would ideally not be part of the implicit price of private schooling. Thanks to Lars Nesheim and Richard Chisik for pointing this out.

[3] The calculated likelihood ratio test statistics are 401.9 for Akron, 944.5 for Cincinnati, 745.4 for Cleveland, 750.3 for Columbus, 767.1 for Dayton, and 405.2 for Toledo.

[4] Standard errors for all non-housing parameter estimates are adjusted for within-group clustering in the house price hedonics and demand estimations, as in Figlio and Lucas (2004).
[5] The implicit price is also adjusted for the tax rate because the property tax affects the capitalization value of education on house prices: households in communities with high tax rates will pay more property taxes and therefore will receive lower capitalization benefits of school quality (Brasington, 2003).
[6] Unlike the inconclusive tests for the house price hedonics, Davidson and MacKinnon's (1981) test for functional form for the demand regression clearly favors the linear functional form. However, the use of LOG TEST SCORE on the right-hand side adds explanatory power over its un-logged counterpart.
[7] Actually, instrumentation may not be necessary when spatial statistics is used (Brasington and Hite, 2004). Endogeneity is a problem of omitted factors being correlated with the regressors. Spatial statistics controls for the influence of omitted variables, thus alleviating the need to treat regressors endogenously. But there is no harm done to the spatial estimation technique by instrumenting (Anselin, 1988).
[8] The central city dummy is chosen to represent lack of choice among public good bundles. The percent living in the community less than six years is chosen to represent people moving in recently, as in Brasington (2002). These variables appear to do their job well: their exclusion creates upward bias on the income parameter estimate as Rubinfeld, et al. (1987) anticipate.
[9] All elasticites are calculated at the mean, unless otherwise noted.
[10] The elasticity of our price with respect to our neighbors' quantity is 0.033.
[11] The sample correlation between the number of bedrooms and income is 0.31.
[12] The sample correlation between income and house price is 0.71.
[13] The 0.43 and 0.82 numbers are from the five-neighbor, proficiency test-based regressions. The same pattern is true for the 200-neighbor regressions and the expenditures-based regressions. The 25th percentile cross-price elasticity for the 200 neighbor regression using test scores for the implicit prices is 0.61 and for the 75th percentile is 1.15. The 25th percentile cross-price elasticity for the regression using expenditures for the implicit prices is 0.51 and for the 75th percentile is 0.97.
[14] See also a theoretical model by Hoyt and Lee (1998) which includes a calculation of the magnitude of the elasticity of private school enrollment necessary for vouchers to reduce taxes.
[15] Many of these programs are discussed in West (1997), Toma (1996), Willms and Echols (1992), and Gauri and Vawda (2003).