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Key Players and Key Groups in Teams *

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Abstract

This paper contributes to the literature on centrality measures in economics by defining a team game and identifying the key players in the game. As an illustration of the theory we create a unique data set from the UEFA Euro 2008 tournament. To capture the interaction between players we create the passing network of each team. This all allows us to identify the key player and key groups of players for both teams in each game. We then use our measure to explain player ratings by experts and their market values. Our measure is significant in explaining expert ratings. We also find that players having higher intercentrality measures, regardless of their field position have significantly higher market values.

Keywords: Social Networks, Team Game, Centrality Measures

JEL Classification Codes: A14, C72, D85

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1 Introduction

In recent years the literature in economics on networks has begun to focus on centrality measures. The seminal paper of Ballester et al. (2006) provides the basic framework to identify key players using centrality measures in networks. However, to the best of our knowledge there is no paper that studies centrality measures using actual data. Our paper makes a contribution to this literature - it modifies the original model of Ballester et al. (2006) to capture team situations by providing the network members a common goal as well. After obtaining the necessary theoretical results we apply it to a unique data set collected from soccer games.

Team like situations dominate many social and economic environments. Firms and organizations are usually made up of smaller groups or teams. Recommendation letters often mention person's ability to be a team player. An applicant's ability to be a team player is also tested in many interviews. Work environments like R&D groups, special task forces and even academia to a certain extent function as teams. Teamwork is an important feature of many games like soccer, basketball and volleyball. This makes understanding the contribution of individual members to a team very useful exercise. It can help design better teams and compensation packages. Identifying the key players in teams is also very useful for retention issues. In this paper, we develop a method for identifying key players and key groups in teams.

There is a substantial literature in graph theory on identifying the key node in a network. To determine the importance of each node, various centrality measures have been developed. These may be degree based measures that take into account the number of links that emanate and end at a node (see for instance Katz (1953), Freeman (1979), Hubbell (1965), Bonacich (1987) and Sade (1989)). Closeness measure like those developed by Sabidussi (1966) and Freeman (1979) use some type of topological distance in the network to identify the key players. Another measure called betweeness measure (see for instance Freeman (1979)) uses the number of paths going through a node to determine its importance. Borgatti and Everett (2006) develop a unified framework to measure the importance of a node. Borgatti (2006) identifies two types of key player problems (KPP). He argues that in KPP-positive situation key players are those who can optimally diffuse something in the network. In a KPP negative situation key players are individuals whose removal leads to maximal disruption in the network.

Ballester, Calvo-Armengol and Zenou (2006, henceforth BCZ) provide microfoundation for the key player problem in fixed networks. Their model considers two vital ingredients: individual actions and interaction between players. In the Nash equilibrium of the game each player chooses their individual action taking both components into account. The key player is the one whose removal leads to the highest overall reduction in effort. Thus, their approach builds strategic behavior into the network, and combines both negative and positive aspects of the problem. Calvo-Armengol, Patacchini and Zenou (2009, henceforth CPZ) extend BCZ (2006) and propose a peer effects model to study educational outcomes.

In this paper, we develop a Team Game based on the individual actions and interactions between players. Additionally, each player gains utility when the team achieves its desired outcome. This team outcome depends on individual effort and an ability term for each player. Another interesting feature is that following BCZ (2006) we define key player problem from a social planner's perspective. In context of teams, team leader or head coaches can be regarded as the social planners. We then develop two new intercentrality measures that takes into account two different criteria for the social planner. The first intercentrality measure is derived form the the reduction in aggregate Nash Equilibrium effort levels whereas the second intercentrality measure is derived using the externality a player gets from her teammates.

This paper has two contributions. The first contribution is providing an empirical illustration of the approach using a team sport, namely soccer. We observe the passing effort of international soccer players to proxy the amount of interaction between players in UEFA European Championship 2008 and identify the key players and key groups in the network. It is important to note that we are not seeking the best player on the field. Rather, we are looking for the player whose contribution to his team is maximal. Finally, we show that players who have higher interactions (passings and receivings) and abilities have significantly higher ratings from experts and market values. We adapt the asymmetries in the interactions which provides a more detailed analysis. Our approach is different from CPZ (2009) in two aspects. CPZ (2009) focuses on peer effects in a student environment whereas we are directly interested in determining key players and ranking the individuals according to their contribution to their teams. The second contribution is extending the BCZ (2006) model and introducing a team (or network) outcome component into the analysis to rank players according to their contributions to their teammates. The Nash equilibrium of the model provides the optimal amount of individual efforts' of each player. It implies that if the player has a higher return for his individual actions or a higher ability parameter, then she will have more incentives to perform individual actions.

Determining a key group instead of a key player is also an interesting aspect since more than one player may have equivalent level of contribution to their teammates. In addition to that, it is important to identify which combination of players have more importance within the network. This information is crucial for the team managers who wish to form a team with individuals who provide different inputs to their teammates. It is important to note that the members of the of key groups are not the best working peers but they are the ones whose joint contribution to their team is maximal. Temurshoev (2008) extends BCZ (2006) paper by introducing the key group dimension. Temurshoev (2008) searches for the key group, whose members are, in general, different from the players with highest individual intercentralities. We apply Temurshoev's (2008) approach to determine the key groups of players.

The approach in this paper can be extended and replicated to more general team situations as well as for other sports where players work in teams. However, in this paper, we provide an empirical example using international soccer matches. Taking this approach has some advantages. First, since soccer is a team sport and the payoff of players depends on the team outcome. Second, interactions within soccer teams are observable and passing effort of players is a good metric to identify these interactions. We create a unique passing data from UEFA European Championship 2008 and identify the key players and key groups of teams which played in the Quarter Final, Semi-Final and Final stage of the tournament.[1]

Subsequent sections of the paper are organized as follows: Section 2 defines the team game and various centrality measures. It also identifies the Nash equilibrium and our team intercentrality measures. Section 3 motivates the use of soccer data as an empirical application. Section 4 identifies the empirical methodology. The paper concludes with discussions and possible extensions.

[1]Fifty European national teams played qualifying stages and only 16 of them were qualified for the UEFA Euro 2008. So, it is reasonable to expect that the quality of the players in the national tournaments are similar. Thus, interaction between players plays a crucial role in determining the outcome of the matches making our results more important.

2 Team Game

In this section, we first define the team game and interpret the model for soccer. Then, we introduce the various centrality measures and find the Nash equilibrium of the game. Finally, we provide the relationship between the Nash equilibrium and the intercentrality measure(s) considering two different scenarios.

We begin by introducing the team game. We define the individual player's payoff function using the notation of BCZ (2006) as far as possible. We also interpret the model variables.

$$U_i(x_1, ..., x_n) = \alpha_i x_i + \frac{1}{2}\sigma_{ii} x_i^2 + \sum_{j \neq i} \sigma_{ij} x_i x_j + \theta Z. \tag{1}$$

The first two terms form a standard quadratic utility function where $x_i \geq 0$ is defined as the individual effort of player i. $\alpha_i > 0$ stands for the coefficient of individual actions and $\sigma_{ii} < 0$, the coefficient of the second term, defines concavity in own effort i.e., $\partial^2 U_i / \partial x_i^2 = \sigma_{ii} < 0$. For simplicity we assume that these coefficients are identical for all players and we drop the subscript.[2]The third term captures the bilateral influences between players with σ_{ij} being the coefficient of this term. Let $\mathbf{\Sigma} = [\sigma_{ij}]$ be the matrix of these coefficients. Note that σ_{ij} could be positive or negative. The last expression is the team outcome term denoting the desired team goal. It represents how an individual's utility depends on team outcome. We assume that the team outcome, Z is a linear function of each player's effort and ability parameter. The coefficient θ is a scale parameter that can be used to capture the importance of the game.

For soccer, x_i term can be interpreted as the attributes such as creativity, distance traveled, attention, speed, shots on goal. α_i measures the returns from individual actions and σ_{ii} introduces the concavity in effort in the sense that as players perform actions, they spend stamina and it becomes costly. We utilize passing behavior to infer the interactions between players. Thus, player i's utility from interacting with player j is weighted by how often he passes to j. σ_{ij} can be interpreted as the complementary action of player i on player j. For the case of soccer, σ_{ij} indicates the number of (discounted) successful passes from player i to j minus the number of (discounted) unsuccessful passes from player i to j.[3] This means that if σ_{ij} is positive, number of successful passes from player

[2]We relax this assumption and consider the cases when α_i and σ_{ii} can be different for every player in Proposition 1 (a)-(b).

[3]We acknowledge that there are other complementarity actions other than passing behavior; however, these factors

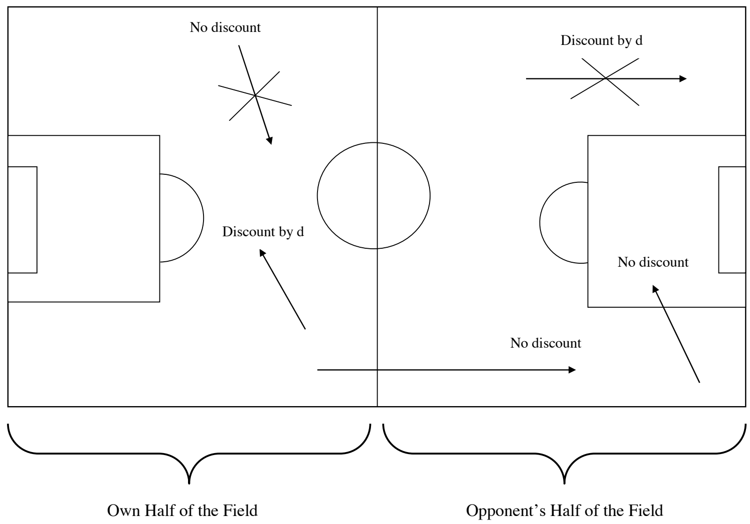

Figure 1: Unsuccessful passing attempts are shown by cross overs.

i to player j exceeds the number of unsuccessful passes from player i to j.

While measuring the complementarity in players' effort, we introduce a discounting parameter, d in constructing the $\mathbf{\Sigma}$ matrix. Passes that are made far from the opponent's goal have little influence on creating a goal scoring opportunity. Therefore, we discount the passes that are made in own half of the field by a factor $0 < d < 1$. On the other hand, if player i successfully passes the ball to player j, and if player j is the opponent's half, then we do not discount that pass. Unsuccessful passes are discounted in the opposite way. If a player i losses the ball while trying to pass to player j, we look at the position of j. If player j is in the opponent's half, then we discount that loss by d. Similarly, if player i losses the ball while trying to pass to player j who is in his own half, then we do not discount that loss. Basically, if player i losses the ball near his own goal then that is a serious loss for the team. The intuition for not discounting the unsuccessful passes made in the own half is that players have to run back which hurts the team's play and may create an opportunity for the opponent to start an attack from an advantageous position. The below figure shows an example of discounting.

For the empirical model, the ability parameter, δ_i is defined as the scoring probability of player i where δ_i = Number of goals scored by player i / Number of total shots on goal of player i.

are very difficult to measure. Taking passing as a metric for complementarity action simplifies the empirical model and enables us to quantify.

Alternatively, Z can also be defined as the outcome of the match. Specifically, Z can be assumed to be taking values of {1, 0, -1} where $Z = 1$ implies that the team wins the game, $Z = 0$ implies that the match ended in a draw, and $Z = -1$ implies that the team lost the match. With the above definition of Z, the Nash equilibrium of the Team Game is identical to the BCZ (2006) and allows us to use the ICM provided by the authors in Remark 5 (pg. 1412). For a soccer game this could be winning the game or scoring more goals. This term contains a the same set of variables for all players since they all share the same outcome. For simplicity, let $Z = \sum_{i=1}^{n} \delta_i x_i$ where δ_i defines each individual's ability to help achieve the team's goal. The parameter θ is a scale factor that could be used to capture the importance of different events for the team.[4]

Our team game differs from that of BCZ (2006) model in the last term. This allows us to consider the n players acting together towards a common objective. While alternative formulations of this are possible, we believe our framework has certain advantages. First, it allows for explicit comparison with BCZ (2006). Second, while all effort by player provides a utility, the effort adjusted by the ability parameter is important for achieving the team outcome. This can be useful for empirical illustration since it may not be possible to obtain data on α_i and σ_{ii}. The ability parameter δ_i on the other hand could be obtained from available data.

In order to proceed following BCZ (2006), we let $\underline{\sigma} = \min\ (\sigma_{ij}|i \neq j)$ and $\overline{\sigma} = \max\ (\sigma_{ij}|i \neq j)$. We assume that $\sigma < \min\ (\underline{\sigma}, 0)$. Let $\gamma = -\min\ \underline{\sigma}, 0 \geq 0$. If efforts are strategic substitutes for some pair of players, then $\underline{\sigma} < 0$ and $\gamma > 0$; otherwise, $\underline{\sigma} \geq 0$ and $\gamma = 0$. Let $\lambda = \overline{\sigma} + \gamma \geq 0$. We assume that $\lambda > 0$. Define $g_{ij} = (\sigma_{ij} + \gamma)/\lambda$. Note that, the g_{ij}'s are weighted and directed allowing us to obtain relative complementarity measures. Consequently, the elements g_{ij} of the weighted adjacency matrix lie between 0 and 1. If we do not use a weighted $\mathbf{G}$ matrix then it contains only 0s and 1s as its elements. This will imply that the additional weight for having more connections with the same player is zero. So, when $g_{ij} = 1$ then there is a connection and if $g_{ij} = 0$ then there is no connection between player i and player j. However, it is very important to identify the relative interaction between players rather than just considering if there is a connection between player i and j. Thus, using a weighted $\mathbf{G}$ matrix is important to illustrate team environments.

The adjacency matrix $\mathbf{G} = [g_{ij}]$ is defined as a zero diagonal nonnegative square matrix. The

[4]In principle, one could define θ_Z to capture the importance of the level of achievement in the team's objective. Here, for simplicity we assume it to be θ.

zero diagonal property assures that no player is connected to themselves (i.e, there are no direct loops from player i to i.) Then, $\mathbf{\Sigma}$ matrix which captures the cross effects can be decomposed into the following expression:

$$\mathbf{\Sigma} = -\beta\mathbf{I} - \gamma\mathbf{U} + \lambda\mathbf{G} \tag{2}$$

where $-\beta\mathbf{I}$ shows the concavity of the payoffs in terms of own actions, $-\gamma\mathbf{U}$ shows the global interaction effect, and $\lambda\mathbf{G}$ shows the complementarity in players' efforts. Using the above decomposition, Equation (1) becomes:

$$U_i(x_1, ..., x_n) = \alpha x_i - \frac{1}{2}(\beta - \gamma)x_i^2 - \gamma\sum_{j=1}^{n} x_i x_j + \lambda\sum_{j=1}^{n} g_{ij}x_i x_j + \theta_z Z \tag{3}$$

for all players $i = 1, ..., n$.

2.1 Centrality Measures

Here, we define the centrality measures needed to identify the key player. Let $\mathbf{M}$ be a matrix defined as follows:

$$\mathbf{M}(\mathbf{g}, a) = [\mathbf{I} - a\mathbf{G}]^{-1} = \sum_{k=0}^{\infty} a^k\mathbf{G}^k. \tag{4}$$

The above matrix keeps track of the number of paths that start from player i and end at player j with a decay factor, a and a given adjacency matrix $\mathbf{G}$. Note that players can also contribute to their teammates through indirect connections, but these have lower weights.

Following BCZ (2006), we define the Bonacich centrality measure as:

$$\mathbf{b}(\mathbf{g}, a) = [\mathbf{I} - a\mathbf{G}]^{-1} \cdot \mathbf{1} \tag{5}$$

where $\mathbf{1}$ is a $n \times 1$ vector of ones, n is number of players in the team and $\mathbf{I}$ is a $n \times n$ identity matrix. The Bonacich centrality measure counts the total number of paths that originates from player i. Note that b_i is the row sum of the $\mathbf{M}$ matrix. Equivalently, the Bonacich centrality measure is $b_i(\mathbf{g}, a) = m_{ii}(\mathbf{g}, a) + \sum_{i \neq j} mij(\mathbf{g}, a)$. Next, we define a weighted Bonacich centrality measure with the ability parameter, δ_i as the weight:

$$\mathbf{b}_\delta(\mathbf{g}, a) = [\mathbf{I} - a\mathbf{G}]^{-1} \cdot \boldsymbol{\delta} \tag{6}$$

We define another centrality measure which accounts for the weighted receivings of the players where the weights are given by δ_i:

$$r_i(\mathbf{g}, a) = \sum_{j=1}^{n} m_{ji}(\mathbf{g}, a) \times \delta_i \tag{7}$$

This (receiving) centrality measure takes into account the paths that end in player i weighted by the ability parameter of the player. This measure captures the externality a player gets from her teammates and weights it according to the ability of the player.

The BCZ (2006) intercentrality measure (ICM) for an asymmetric $\mathbf{G}$[5] is given by:

$$\tilde{c}_i(\mathbf{g}, a) = b_i(\mathbf{g}, a) \times \frac{\sum_{j=1}^{n} m_{ji}(\mathbf{g}, a)}{m_{ii}(\mathbf{g}, a)} \tag{8}$$

Unlike the Bonacich centrality measure, ICM takes into account both the connections that player i sends to her teammates and the number of connections that player i receives.

Through the paper, we define two intercentrality measures to take into account possible different objectives of the social planner while identifying the key player of the team. We provide two alternative objectives for the social planner. In the first case, the social planner determines the player whose removal leads to the highest amount of reduction in the aggregate Nash Equilibrium effort. We derive the intercentrality measure for this objective and call this measure as team intercentrality measure, $TICM$. For an asymmetric $\mathbf{G}$ matrix, we define $TICM$ as:

$$\bar{c}_i(\mathbf{g}, a) = b_i(\mathbf{g}, \lambda^*) \times \left(\frac{\sum_{j=1}^{n} m_{ji}(\mathbf{g}, \lambda^*) + \sum_{j=1}^{n} m_{ji}(\mathbf{g}, \lambda^*)\delta_j}{m_{ii}(\mathbf{g}, \lambda^*)}\right) \tag{9}$$

$TICM$ measures player i's contribution to the interaction matrix as well as her contribution to the team outcome. The difference between ICM and $TICM$ is in the last term in the parentheses which captures player i's importance in creating the team outcome.

Alternatively, one can argue that the social cares both about the interaction between players as well as the externality term in the payoff of each player. While the $TICM$ measure above takes the first effect into account it does not take the second effect into account. Therefore, we introduce a second measure of the importance of a player in the game by taking the interaction into account as well as how the contribution of other players affects the performance of each player weighted by their ability. We derive the intercentrality measure for this objective and call this measure as

[5] Note that, in the context of teams, the $\mathbf{\Sigma}$ matrix is unlikely to be symmetric since the number of paths from player i to player j will be different for at least one pair. Hence, an asymmetric $\mathbf{\Sigma}$ matrix will lead to an asymmetric $\mathbf{G}$ matrix.

team intercentrality measure with externality, ($TICM^e$). For an asymmetric $\mathbf{G}$ matrix, we define $TICM^e$ as:

$$\hat{c}_i(\mathbf{g}, a) = b_i(\mathbf{g}, a) \times \frac{\sum_{j=1}^{n} m_{ji}(\mathbf{g}, a)}{m_{ii}(\mathbf{g}, a)} + \sum_{j=1}^{n} m_{ji}(\mathbf{g}, a) \times \delta_i \tag{10}$$

The primary difference between ICM and $TICM^e$ is in the last term which measures the externality player i receives from her teammates weighted by the ability parameter of the player.

2.2 Nash Equilibrium of the Team Game

In this section, we show that the Team Game has a unique interior Nash equilibrium by the following theorem.

Theorem 2.1 Consider a matrix of cross-effects which can be decomposed into (3). Suppose $\sigma_{ij} \neq \sigma_{ji}$ for at least one $j \neq i$, $\beta/\lambda > (\rho(\mathbf{G}))$ and a small enough θ. Define $\lambda^* = \lambda/\beta$. Then, there exists a unique, interior Nash Equilibrium of the team game given by:

$$\mathbf{x}^*(\mathbf{\Sigma}) = \frac{\alpha \mathbf{b}(\mathbf{g}, \lambda^*) + \theta \mathbf{b}_\delta(\mathbf{g}, \lambda^*)}{\beta + \gamma \hat{b}(\mathbf{g}, \lambda^*)}$$

where $\hat{b}(\mathbf{g}, \lambda^*) = \sum_{i=1}^{n} b_i(\mathbf{g}, \lambda^*)$.

Proof: The proof is an adaptation of BCZ (2006) and can be found in Appendix.

The Nash equilibrium of the Team Game has some interesting implications. First, it identifies the optimal effort of individuals in the network based on the given interactions between players. It also explains why some players provide higher individual effort by indicating that players who have higher ability parameter or who make more interactions with their teammates, will have higher individual effort.

A unique interior Nash equilibrium exists even when players have heterogeneity in returns (α_i) and concavity (σ_{ii}) in individual actions are proved in Proposition 1 (a) and (b).

Proposition 1 (a): Suppose $\alpha_i \neq \alpha_j$ and θ is small enough then Nash equilibrium of the Team Game is:

$$\boldsymbol{x}^*(\mathbf{\Sigma}) = \frac{\mathbf{b}_\alpha(\mathbf{g}, \lambda^*) + \theta \mathbf{b}_\delta(\mathbf{g}, \lambda^*)}{\beta + \gamma \mathbf{b}(\mathbf{g}, \lambda^*)}$$

(b): Suppose $\alpha_i \neq \alpha_j$, $\sigma_{ii} \neq \sigma_{jj}$ for at least one player and θ is small enough, then Nash equilibrium of the team game is:

$$\boldsymbol{x}^*(\mathbf{\Sigma}) = \frac{\mathbf{b}_{\widetilde{\alpha}}(\mathbf{g}, \widetilde{\lambda}^*) + \theta \widetilde{\mathbf{b}_\delta}(\mathbf{g}, \lambda^*)}{\widetilde{\beta} + \widetilde{\gamma} \hat{\mathbf{b}}(\mathbf{g}, \widetilde{\lambda}^*)}$$

Proof: See Appendix.

2.3 Key Player

In this subsection, we develop two alternative measures of identifying the key player in the teams by considering different criteria of the social planner. In the first case, social planer is interested in finding the player whose removal causes highest amount of reduction in the aggregate Nash Equilibrium. In this approach, we determine the key player as in BCZ (2006) which is to minimize sum of efforts after removal. We denote by $\mathbf{G}^{-i}$ (resp. $\mathbf{\Sigma^{-i}}$) the new adjacency matrix (resp. matrix of cross-effects), obtained from $\mathbf{G}$ (resp. from $\mathbf{\Sigma}$) by setting all of its i^{th} row and column coefficients to zero. The resulting network is $\boldsymbol{g^{-i}}$. The social planner's objective is to reduce $\mathbf{x}^*(\mathbf{\Sigma})$ optimally by picking the appropriate player from the population. Formally, she solves max $\{\mathbf{x}^*(\mathbf{\Sigma}) - \mathbf{x}^*(\mathbf{\Sigma}^{-i})|i = 1, ..., n\}$. This is a finite optimization problem and has at least one solution. Theorem 2.2 shows that the player who has the highest amount of $TICM$ will be the solution of this problem.

Theorem 2.2 Let $\beta > \lambda\rho_1(\mathbf{G})$. The key player of the Team Game, i^* solves $max\{\mathbf{x}^*(\mathbf{\Sigma}) - \mathbf{x}^*(\mathbf{\Sigma}^{-i})|i = 1, ..., n\}$ and has the highest team intercentrality measure ($TICM$) in $\mathbf{g}$, that is $\bar{c_{i^*}}(\mathbf{g}, \lambda^*) > \bar{c}_i(\mathbf{g}, \lambda^*)$ for all $i = 1, ..., n$.

Proof: See Appendix

Alternatively, social planner would like to consider the externalities players get from their teammates which is not included in the Nash Equilibrium of the team game. It might be the case that social planner is interested in considering each player's effect on the interaction matrix as well as taking the externality into account. BCZ (2006) provides the effect of player i's removal to the interaction matrix under Remark 5 for the asymmetric case. We define the externality player i receives from her teammates $r_i(\mathbf{g}, \lambda^*) = \sum_{j \neq i} m_{ji}(\mathbf{g}, \lambda^*)$ and we weight that with the ability parameter of the player. Theorem 2.3 indicates that the player who has the highest amount of $TICM^e$ will be the solution of this case.

Theorem 2.3 Let $\beta > \lambda\rho_1(\mathbf{G})$. The key player of the Team Game with externalities, i^* solves $b(\mathbf{g}, \lambda^*) - b(\mathbf{g}^{-i}, \lambda^*) + r^i_\delta(\mathbf{g}, \lambda^*)$ and has the highest $TICM^e$ in $\mathbf{g}$, that is $\hat{c_{i^*}}(\mathbf{g}, \lambda^*) > \hat{c}_i(\mathbf{g}, \lambda^*)$ for all $i = 1, ..., n$.

Proof: See Appendix

3 Soccer: A Team Game and The Role of Passing

Modern soccer is very much a team game. The performance of players depends crucially on each other's actions and interaction between players forms a vital component of the game. Soccer coaches, training books and authorities emphasize the team aspect of the game. As the great Brazilian soccer player Pele said in a press conference in Singapore in November 2006, "I think the problem with Brazil was lack of teamwork because everybody used to say Brazil will be in the final." Pele added that Brazil had the best individual players against France, but they lost the game because they could not play as a team.[6] On November 29, 2007, Gerard Houllier, the famous technical director of the French Football Federation, speaking at the 9th UEFA Elite Youth Football Conference summed this up as "Teamwork is the crux of everything."[7]

One important aspect of soccer that makes it a team game is the fact that passing is a very crucial part of the game. In the early days of soccer, the game was based on individual skills such as tackling and dribbling. In 1870s, the Scots invented the passing game and everyone soon realized that it is easier to move the ball than players since the ball travels faster than humans. Since then passing and receiving have become a key part of a soccer team's strategies. A soccer training manual by Luxbacher (2005) emphasizes the importance of passing in the following "Passing and receiving skills form the vital thread that allows 11 individuals to play as one - that is the whole to perform greater than the sum of its parts." Similarly, Miller and Wingert (1975) addresses the importance of passing in soccer by stating that "There are no more crucial skills than passing in soccer because soccer is a team sport. The most effective set plays involve accurately passing and receiving the ball."

Luhtanen et al. (2001) report that successful passes at the team level are important for explaining the success in the UEFA European Championship 2000. Specifically, Luhtanen et al. (2001) document that there is one to one relationship between the ranking of the team in Euro 2000 and the ranking of the team in terms of successful passing and receivings. Thus, it seems reasonable that passing is a good metric for identifying the interactions between players.

Figure 2 displays the relationship between average number of shots per game and average num-

[6]See http :// findarticles.com/p/articles/mi kmafp/is 200611/ai n16939060.

[7]See http://www.uefa.com/uefa/keytopics/kind=1024/newsid=629284.html

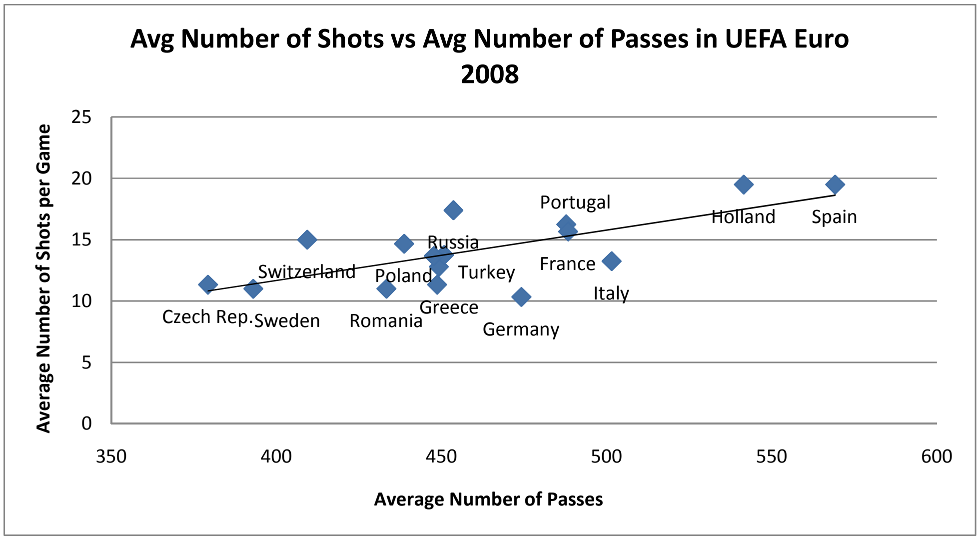

Figure 2: The relationship between average number of shots per game and average number of passes per game in UEFA Euro 2008.

ber of passes per game of the national teams in the UEFA European Championship 2008.[8] The correlation coefficient between these variables is 0.7. The regression coefficient obtained from regressing average number of shots on goal per game on average number of passes per game indicates that on the average 27 passes created 1 additional shot on goal for the team in Euro 2008. This is consistent with the idea that teams need ball possession to create goal scoring opportunities which directly affects the outcome of the match. Clearly, passing is an important interaction variable in our dataset.

There are some advantages to using passing to infer player interactions. First, it is pairwise and both the originator and receiver must be successful to complete the action. The pairwise aspect of passing enables us to utilize the network theory to understand the contribution of each player to the team. Second, passing as a measure of interaction is observable and easily quantifiable. Data for other aspects of the soccer such as tackling, dribbling or off the ball movement of players are very hard to observe. In addition, often identifying the quality of these actions require subjective judgement. Finally, even if we had data about these aspects, it would be still difficult to quantify

[8]This data was accessed from the following website http://www1.uefa.com/tournament/statistics/teams. It is available from the authors upon request.

those variables exactly.

4 Empirical Methodology

This section illustrates our methodology for identifying the key player and key groups in soccer teams. First, we describe our data collection process. Next, we calculate the ICM and $TICM^e$ by using the corresponding definitions in the paper and provide our results for the key players and key groups. Finally, we conduct sensitivity checks for the model parameters which are used for identifying key players.

4.1 Data and Results

Our data consists of all the matches from the Quarter Final onwards for the UEFA European Championship 2008. All the data that is used in the paper is available from the authors on request. Unfortunately, official passing data from UEFA's website is not adequate for our study due to a number of reasons. First, UEFA provides data only on the successful passes between player i and j and excludes the unsuccessful passes. Second, UEFA statistics do not provide the passing position of the players which is important for assessing the quality of passing. Hence, we created a unique passing data set ourselves by watching the matches from DVD's. This was done by freezing the frame at the time of the passing attempt and recording the player making the pass and the receiver in a matrix by noting the position of the receiver. We also discounted the passes using the method described in the previous section.[9] The net discounted passes are used to determine the $\sigma'_{ij}s$ in $\mathbf{\Sigma}$ matrix. As expected, the $\mathbf{\Sigma}$ and $\mathbf{G}$ matrices are both asymmetric.

In order to facilitate comparisons across matches, we define a tournament wide λ and γ which are the same for every team. First we obtain the highest amount of positive and negative interaction between each pair of players throughout the tournament. Using this, the tournament wide γ and λ parameters are chosen as 5 and 20 respectively. This allows us to compare the same player's intercentrality measure from different matches as well as to compare the intercentrality measure of different players from different matches.[10]

[9] According to our passing data on average 972.5 successful passes occurred in a match and it is a tedious exercise to record every passing attempt. Note that we also take into account the unsuccessful passing effort which are 272.3 on average in each match.

[10] Note that the Netherlands vs Russia, Spain vs Italy and Croatia vs Turkey matches went into the extra time. Therefore, comparing the players in the these games with those ended in 90 minutes is not possible. The cross

Data for creating the tournament wide scoring probabilities of each player was obtained from ESPN's website.[11] Ideally, the life time scoring probability of a player would be δ_i in the model. However, this data is not available and we use the tournament wide measure as a proxy for this. Next, we calculate the $\mathbf{M}$ matrix, centrality vector ($\mathbf{b}$) and (team) intercentrality vectors $\mathbf{c}$ and $\hat{\mathbf{c}}$ by using the definitions provided in Section 2. Note that assigning a value to a is crucial for obtaining a pure and interior Nash equilibrium. BCZ (2006) note that for the case of asymmetric $\mathbf{\Sigma}$ and $\mathbf{G}$ matrices, a should be less than the spectral radius of $\mathbf{G}$, which is inverse of the norm of the highest eigenvalue of $\mathbf{G}$. The greatest eigenvalue of $\mathbf{G}$ matrices for the teams in the sample is 7.07 and hence following the above rule, the decay factor, a, is set to 0.125 for all matches. Since we did not have any guide lines for discount factor, $d \in [0, 1]$ we assume that $d = 0.5$ for all matches. Using all of these parameters we then compute ICM and $TICM^e$ of each player. To make comparisons, we also provide some results for $TICM$.

The corresponding calculations for the Final, Semi Final and Quarter Final games for Euro 2008 are reported for each team in Tables 1-7. In those tables, $\hat{\mathbf{c}}$ refers to $TICM^e$ and $\mathbf{c}$ indicates ICM of BCZ (2006). We find that the results obtained by using $TICM^e$ are generally better at capturing the players who have a direct influence on the outcome of the matches since it also incorporates the scoring probabilities. The highest value of $TICM^e$ is observed in the Spain vs Italy Quarter Final game for Fabregas who has a value of 8.43. Note that this match ended in extra time. In all of the matches which ended in normal time, the highest value of $TICM^e$ is observed in the Germany vs Portugal Quarter Final game for Deco of Portugal who has a value of 6.44. The highest ICM reported as 7.97 in Spain vs Italy match for David Silva of Spain. Note that this match ended in extra time. The next highest ICM is in Germany vs Portugal match (which ended in normal time) for Deco 5.81. Unlike the conventional belief that midfielders would always be key players due to their field position, there are examples in our data that proves otherwise. For instance, in Netherlands vs Russia match, Russian key player turns out to be a Arshavin, a forward player. Also, the major difference $TICM^e$ and ICM is that forward and midfield players appear in the higher ranks according to $TICM^e$. In Tables 8 and 9, we provide all the intercentrality measures($TICM$, $TICM^e$ and ICM) for Spain vs Germany Final and Netherlands vs Russia Quarter Final matches. The

comparisons are valid for match lengths of the same duration. We discuss this issue in more detail in the next section.

[11] http://soccernet.espn.go.com/euro2008/stats

difference between $TICM$ and ICM provides the externality player i receives from his teammates. The difference between $TICM$ and ICM yields the contribution of the player to the team outcome. It can be seen in all tables that if $\delta_i = 0$, then $TICM^e$ and ICM are equal to each other. However, $TICM$ can still be different since it includes δ_j's.

An important fact to mention is that since the data on scoring probabilities of players is not life time scoring probabilities, $TIMC$ and $TICM^e$ cause players who have very few shots in the tournament but scored a goal to have a high measures in some matches. Therefore, we report ICM results as a sensitivity check.[12] Some teams in our data are observed more than once and yet the key player in the same team differs in different matches. This might be due to the fluctuations in the performance of players as well as the different playing style of the players in different matches.

For the case of soccer, it is mostly the case that the head coach of the national team as the social planner would not be very interested in the reduction in aggregate Nash equilibrium. There is no guarantee that teams having more effort or playing better will get higher ranks in the tournament. Hence, we will use the externality scenario since it includes the quality of effort of the players directly and it captures the externality a player gets from the social planner's perspective, not through the player optimization through the Nash Equilibrium. We provide some calculations of $TICM$ in the next section to make some comparisons however, through the empirical part, we mainly focus on $TICM^e$ and ICM.

4.2 Sensitivity Checks

There is a concern that determination of the key player may depend on the our chosen values of the decay factor, a and discount factor, d. In fact, by means of an example BCZ (2006) show that the key player may be different for different values of a. Similarly, the key player may change depending the value of discount factor, d. Hence, in order to check the robustness of our results, we conduct a simulation analysis by changing the values of those parameters. We allow a to vary from 0 to 0.125 in increments of 0.001. Simultaneously, we use the same increment and increase the value of d from 0 to 1. Since we perform the simulations for all matches and all teams, this gives us 14 $\times$ 125,000 = 1.75 million simulations. We find that the key player identified by ICM changes about 15 percent of

[12]According to our data there are a few players such as Van Bronchorst of Netherlands and Lahm of Germany who has only one or two shots on goal through the tournament however scoring a goal in those attempts. That makes their scoring probability relatively higher and substantially increases their $TICM^e$.

the simulations. On the other hand, the identified key players by using $TICM^e$ change 40 percent of the simulations. There is a greater variability in $TICM^e$ results since the scoring probability of the players are specific to the Euro 2008 tournament. Since the scoring probability itself shows great variability, it makes the $TICM^e$ measure more idiosyncratic. The passing game on the other hand is more stable and therefore the ICM results have smaller variation.

4.3 Key Group

In this section, we determine the key groups of players. The idea of searching for the key group was initiated by BCZ (2004). However, in this paper we prefer to follow Temurshoev's (2008) approach for computational convenience. Key groups of players in the matches provide information about the joint performance of players in the group. This is a valuable information for the soccer clubs, managers and coaches who wish the form their teams with individuals that provide different adjacencies to their teammates.[13] In order to identify key groups of size k in a team, we take every possible combination of k players from the team and determine the reduction in the interaction matrix as well as the externalities. The key group consists of players whose joint removal leads to largest reduction.

We use Temurshoev's (2008) approach to compute the TICM of a group of k players. Removing players from the game causes a reduction in the interaction between players in addition to the reduction to the loss of those players ability. Therefore, we derive the group intercentrality measure for $TICM^e$ as:

$$\hat{c_{\boldsymbol{g}}} = \mathbf{b}'\mathbf{E}(\mathbf{E}'\mathbf{M}\mathbf{E})^{-1}\mathbf{E}'\mathbf{b} + (\mathbf{1}'\mathbf{M}\mathbf{E})(\mathbf{E}'\boldsymbol{\delta}) \tag{11}$$

where $\mathbf{E}$ is the $n \times k$ matrix defined as $\mathbf{E} = (e_{i1}, ..., e_{ik})$ with e_{ir} being the i_r^{th} column of the identity matrix, k being the number of players in the group and $1 \leq k \leq n$.

The first term captures the effect of the removal of a group of players in $\mathbf{g}$ and the second term captures the effect of reduction in the desired outcome of the team. It can be readily checked that for $k = 1$, the above expression boils down to the team intercentrality measure with externality ($TICM^e$) of a player which is given in Equation (10). Note that the key group is not always comprised of the individuals having the highest intercentrality measure. As described in Borgatti

[13]The identified key groups do not reveal best working individuals, but it reveals which combination of players have higher importance in the interactions and externalitites.

(2006) and Temurshoev (2008), according to the *redundancy principle* key group involves players who provide different adjacency to their teammates.

We choose key group sizes of $k = 2$ and $k = 3$ and calculate every possible group's intercentrality measure using Equation (11). The key group results for all the countries and matches in the sample are provided in Tables 10-14. In these tables, we report the top two (the best and the next best) key groups. In the key group tables, the column player position identifies the field position of the player. These positions are D (Defense), M (Midfield) and F (Forward). The rank in the $\hat{\boldsymbol{c}}$ column identifies the player's rank according to (TICM).

For an interesting comparison, we also provide the ICM key group results of Spain in Table 15.[14] Generally, the key groups obtained by using $TICM^e$ include more forwards and midfielders. According to Table 15, there are no forward players and several defenders in key groups. However, according to Table 10, key groups according to $TICM^e$ have some forwards and more midfielders.

4.4 Player Ratings, Market Value and (Team) Intercentrality

In this subsection, we discuss the effect of the ICM and $TICM^e$ on player ratings and market values of the players in our sample.

4.4.1 Player Ratings

We consider player ratings by experts after each game to show that the individual performance of the players can be explained by the (team) intercentrality measures. We obtain player ratings from three sources: Goal.com, ESPN and SkySports. We create a variable called rating for each player which is obtained by taking of the average of these ratings.[15] These sources are used since they use the same scale and also provide ratings for the substitute and substituted players in the matches. Also, these sources are outside the competing countries in UEFA Euro 2008 which eliminates potential country bias in the ratings.

In order to analyze the relationship between player ratings and ICM and $TICM^e$, we consider

[14] The key groups according to ICM for other countries are available upon request.

[15] The correlation coefficient of ratings from the above sources are 0.7 thus we prefer to take the average of these ratings rather than using them one by one.

the following base model:

$$\text{Rating}_{it} = \alpha_1 + \beta_1\ ICM_{it}(TICM^e_{it}) + \gamma_1\ Age_i + \theta_1\ Age_i^2 + \lambda_1\ Position_i + \psi_1\ ClubRank_i + \phi_1\ NationRank_i + \epsilon_{it}.$$

In the above regression model, the i subscript represents the player i and the t subscript represents the match t. Rating is the dependent variable and represents for the average of the player ratings obtained from the three sources. ICM stands for the intercentrality measure of BCZ (2006). $TICM^e$ represents the team intercentrality measure with externalities from Equation (10). Position is a dummy variable that identifies the field position of the player. We consider three different field positions: Defense (D), Midfield (M) and Forward(F).[16] The *transfermarkt.de* website provides information about other observable characteristics of the players such as: Date of birth, club, nation, position, and number of international appearances, number of international goals, preferred foot and captaincy. We use the Club UEFA points and Nation UEFA points which are available from UEFA's website in order to capture quality and reputation of the players. Club and Nation points are announced by UEFA yearly. These points are earned for being successful in UEFA club or national tournaments. The points that are provided by UEFA for the year 2008 are composed of the points earned in 2003-2008 period. We merge the available data from *transfermarkt.de* with the (team) intercentrality measure, Club and Nation Rank measured by the UEFA points in 2008. The descriptive statistics about the data set are provided in Table 16.

The estimation results for the relationship between average player ratings, ICM and $TICM^e$ are provided in Tables 17 and 18. In the pooled OLS estimation, we estimate a linear regression model where the time variable, t which is used to identify each match.[17] Table 18 reports the results from a GLS estimation with bootstrapped robust standard errors. We take the average of the ratings and (team) intercentrality measures and have only one observation for each player and we report robust standard errors. Both Tables 17 and 18 show that there is a strong relationship between the $TICM^e$ and the average ratings. Specifically, players who have higher $TICM^e$ performed better than their teammates according to the experts. In the pooled OLS estimation, the estimated coefficient for the

[16]Goalkeepers are excluded from the regression analysis. Niko Kovac (Croatia) retired from professional soccer before 2010 and are also excluded.

[17]Ideally, we would prefer to run a random effects model, but there is not enough idiosyncratic variance in the data to allow for this.

$TICM^e$ is significant at 5 % significance level while the coefficient of ICM is significant at 10 % significance level. In the GLS estimation, the estimated coefficient of $TICM^e$ is significant at 1 % significance level while the estimated coefficient of ICM is significant at 5 % significance level. As a sensitivity check, we only include the players who played longer than 30 minutes in the matches. This reduces the number of observations by 30, but the results are robust. Another important factor to control for is whether or not the match ended in normal time. We define a dummy variable ET which is equal to 1 if the match ended in extra time and 0 otherwise. With the inclusion of this variable, the estimated coefficient of $TICM^e$ is significant at 5 % significance level whereas the estimated coefficient of ICM is not statistically different from zero. Therefore, we conclude that $TICM^e$ and ICM explain the expert ratings.

In order to investigate whether the experts regard different importance to the ICM and $TICM^e$ according to their field position of the players, we interact the ICM and $TICM^e$ of players with their position dummies. None of the estimated coefficients are statistically significant. Therefore, we conclude that the ICM and $TICM^e$ are equally important regardless of the field positions of the players. (i.e, the effect of the intercentrality measures on ratings is homogenous in the sample with respect to players' positions on the field.) In addition to the control variables in base model, we run regressions with a broader set of control variables including international appearances, international goals, captaincy, height and preferred foot. The estimated coefficients and their significance are very similar.

4.4.2 Market Values

Next, we investigate whether having a higher ICM or $TICM^e$ in Euro 2008 affects the market values of the players. Investigating the effect of intercentrality on salaries would be more interesting, but the club salaries of soccer players in Europe are not publicly available. Hence, we consider market values instead of salaries. Frick (2007) and Battre et al. (2008) regard the estimated market value of the soccer players obtained from http://www.transfermarkt.de [18] as a good and reliable source to proxy the undisclosed salary of players. Battre et al.(2008) points out that there is a strong relationship between the market value of the players and their salaries for the players in Bundesliga,

[18]transfermarkt.de does not allow user to track the past market values. We saved the data about the players at March, 19 2010.

German First Division.[19] So, the estimated market value of the players are obtained for the year 2010 may be regarded as a proxy for the salaries of the players in our sample.

We use the following base model to investigate the relationship between the market values of soccer players and their ICM or $TICM^e$:

$$\begin{aligned} LogMV_i &= \alpha_2 + \beta_2\ ICM_i(TICM_i^e) + \gamma_2\ Age_i + \theta_2\ Age_i^2 + \lambda_2\ Position_i \\ &+ \psi_2\ ClubRank_i + \phi_2\ NationRank_i + u_i. \end{aligned}$$

In the above model, Log MV is the dependent variable obtained from *transfermarkt.de* and represents the log of the market value of the players in million euros. Another important factor affecting the market values of players might be the contract length of the players because of the Bosman Rules in European football. It is likely that players whose contracts are about to expire have lower market values.[20] Using the contract duration information available from *transfermarkt.de* we identify the players whose contracts' are expiring at the end of 2009-2010 season. Inclusion or exclusion of those players do not affect our results. However, we find evidence that players whose contracts are expiring in 2009-2010 season have lower market values. Another factor to control for is whether or not the player transferred to another club between 2008 and 2010. We define a dummy variable called move and it is equal to 1 if the player has complected a transfer and 0 otherwise. The estimated coefficient and significance of ICM and $TICM^e$ is robust.

Some players are observed more than once in the tournament and they have different average ratings and (team) intercentrality measures in different matches. However, we have only one observation for the market value of the players and the other control variables are time independent with the current setup. Thus, the above model cannot be estimated by panel data methods. In order to deal with this issue, we take the average of the (team) intercentrality measures and use GLS estimation with bootstrapped robust standard errors. We also run sensitivity regressions with clustered errors according to players, the results are very similar.

The estimation results investigating the relationship between the estimated market value and

[19] Battre et al.(2008) obtains estimated market values of soccer players from a German sports magazine Kicker. However, Kicker only provides the market values of the players who only play at Bundesliga. They conduct a sensitivity check with transfermarkt.de data and they state that the correlation between those two sources are high.

[20] Bosman Rules is an important factor affecting the free movement of labor and had a profound effect on the transfers of football players within the European Union (EU). It allows professional football players in the EU to move freely to another club at the end of their contract with their present team.

(team) intercentrality measures in Euro 2008 is provided in Table 19. We again report the results for both ICM and $TICM^e$. The standard errors are bootstrapped with 1000 replications.[21]

According to the estimation results, intercentrality measure in UEFA Euro 2008 explains the 2010 market values of the players. One standard deviation increase in ICM creates on the average 15.62 percent increase in the market values of the players. On the other hand, one standard deviation increase in $TICM^e$ yields on the average 18.22 percent increase in the market values of the players. It might be the case that, intercentrality measures are important for only a certain group of players who play in a certain position of the field (say midfielders). To test this hypothesis, we interact the ICM and $TICM^e$ of players with their position dummies. The findings suggest that ICM and $TICM^e$ are equally important at 5 % significance level. (i.e, the effect of the intercentrality measures on the market values is homogenous in the sample with respect to players' positions on the field.) In addition to the control variables in the above model, we regress the same dependent variable on a broader set of control variables including national team dummies, international appearances, international goals, captaincy, height and preferred foot. The estimates are close and the coefficient of ICM and $TICM^e$ variables are still significant. Since we have a small sample size, we prefer to use and report the results for the base models.

Note that the regression models use the intercentrality measures (ICM and $TICM^e$) which are calculated for specific parameters of $a = 0.125$ and $d = 0.5$. As a sensitivity check, we calculated those intercentrality measures for the parameter sets $a = 0.1$ and $d = 0.4, 0.5, 0.6$ and $a = 0.125$ and $d = 0.4, 0.6$. The estimated coefficients and their significance are very similar.[22]

5 Conclusion and Discussion

In this paper, we introduce a Team Game and develop a measure for identifying the key player in teams. Our work extends the intercentrality measure of BCZ (2006) to include an additional term which captures the team outcome expression in the utility functions of players. This term suggests that a player gets utility when her team achieves its desired outcome. To identify the contribution of players to their teammates, we develop two intercentrality measures which derives

[21]Since we have only one market value observation for the players, we lose significant amount of observations. To deal with this issue, we bootstrap the standard errors.

[22]We do not report the estimates obtained by using the above parameters but they are available upon request.

from possible considerations of the social planner. $TICM$ considers the effect of a player's removal on the aggregate Nash Equilibrium effort levels. $TICM^e$ identifies the externality each player gets from her teammates and weights it according to the ability of the player.

Our measures also have some common features with intercentrality measure (ICM) of BCZ (2006). We can say that a key player does not need to have the highest amount of individual payoff. In addition, a key player does not need to have the highest amount of individual action. It is important to note that both BCZ (2006) and our framework are not seeking the best players in the network. The identified key players and key groups are the ones that have the highest contribution to the corresponding aggregate Nash equilibrium effort levels or according to the externality scenario the key players are the ones who get the highest amount of externality from their teammates which is weighted by the ability parameter of each player.

In the empirical part of the paper, first we illustrate how to utilize the intercentrality measures. Then, we show that there is a positive relationship between the average ratings and $TICM^e$ and ICM in the sample. This fact reflects that soccer players having more interactions with their teammates get more credit in performance by the experts. Moreover, the market value of the soccer players increase with both $TICM^e$ and ICM which is assumed to be reflected in their salaries. This effect is homogenous in the sample, it doesn't depend on the position of the player on the field.

One interesting extension of the approach in the paper might be considering the effort variable to be a vector and allowing different types of individual actions. This will require a new set of theocratical results. Depending on the availability of data this model then can be empirically tested. In soccer, for instance one could include distance traveled, tackling and dribbling data. Given the relationship between passing and scoring opportunities, this way will not alter our primary results, but will provide us a more precise way to identify key players and key groups.

An interesting extension to our model would be to investigate key player problem as a network design game. The planner is the head coaches who have to announce the national squads. There are qualities, δ_i's and possible interaction possibilities between players. This can be modeled as an expected utility maximization problem with a two stage team game. At the first stage, squads are announced and at the second stage players optimize their effort with given interactions.

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7 Appendix

Proof of **Theorem 2.1**

The condition for a well defined interior Nash equilibrium of the Team Game is that the $[\beta\mathbf{I}-\lambda\mathbf{G}]^{-1}$ matrix must be invertible. We can rewrite the $[\beta\mathbf{I}-\lambda\mathbf{G}]^{-1}$ matrix as

$$\lambda[\frac{\beta}{\lambda}\mathbf{I}-\mathbf{G}]^{-1}$$

Let $(\rho_1(\mathbf{G}))$ be the spectral radius of $\mathbf{G}$ matrix.[23] Then, $\beta > \lambda(\rho_1(\mathbf{G}))$ ensures that Equation (9) is invertible by Theorem III of Debreu and Herstein (1953, pg.601). Once the condition is verified, an interior Nash equilibrium in pure strategies $\mathbf{x}^* \in R^n_+$ satisfies:

$$\frac{\partial U_i}{\partial x_i}(x_i^*) = 0 \quad \text{ and } x_i^* > 0 \quad \text{ for all i=1, 2,...,n}$$

Hence, maximizing U_i with respect to x_i yields:

$$\begin{aligned}\frac{\partial U_i}{\partial x_i} &= \alpha_i + \sigma_{ii}x_i + \sum_{j\neq i}\sigma_{ij}x_j + \theta\delta_i = \mathbf{0} \\ &= \alpha - \beta x_i - \gamma\sum_{j\neq i}^{n} x_j + \lambda\sum_{i=1}^{n} g_{ij}x_j + \theta\delta_i\end{aligned}$$

In vector notation:

$$\frac{\partial \boldsymbol{U}}{\partial \boldsymbol{x}} = \alpha\mathbf{1} - (\beta\boldsymbol{I} - \gamma\boldsymbol{U} - \lambda\boldsymbol{G})\mathbf{x} + \theta\boldsymbol{\delta} = \mathbf{0} \tag{12}$$

The above equation can be rewritten as:

$$\beta(\boldsymbol{I} - \lambda/\beta\boldsymbol{G})\mathbf{x}^* = \alpha\mathbf{1} - \gamma\boldsymbol{U}\mathbf{x}^* + \theta\boldsymbol{\delta}$$

Let $\mathbf{x}^*(\boldsymbol{\Sigma})$ be the solution to the above equation. By using $\boldsymbol{U}\mathbf{x}^* = \hat{x^*}\mathbf{1}$ where $\hat{x^*} = \sum_{i=1}^n x_i^*$ and rearranging terms we obtain:

$$\beta(\mathbf{I} - \lambda^*\mathbf{G})\mathbf{x}^* = (\alpha - \gamma\hat{x^*})\mathbf{1} + \theta\boldsymbol{\delta}$$

Multiplying both sides with $(\mathbf{I} - \lambda^*\mathbf{G})^{-1}$ yields:

$$\beta\mathbf{x}^* = (\alpha - \gamma\hat{x^*})(\mathbf{I} - \lambda^*\mathbf{G})^{-1}\mathbf{1} + \theta(\mathbf{I} - \lambda^*\mathbf{G})^{-1}\boldsymbol{\delta}$$

[23]Spectral radius of $\mathbf{G}$ matrix is defined as the inverse of the norm of the highest eigenvalue of $\mathbf{G}$ matrix.

By using the definitions $\mathbf{b}(\mathbf{g},\lambda^*) = (\mathbf{I}-\lambda^*\mathbf{G})^{-1}\mathbf{1}$ and $\mathbf{b}_\delta(\mathbf{g},\lambda^*) = (\mathbf{I}-\lambda^*\mathbf{G})^{-1}\boldsymbol{\delta}$ the above expression becomes:

$$\begin{aligned}\beta\mathbf{x}^* &= (\alpha - \gamma\hat{x^*})\mathbf{b}(\mathbf{g},\lambda^*) + \theta\mathbf{b}_\delta(\mathbf{g},\lambda^*) \\ &= \alpha\mathbf{b}(\mathbf{g},\lambda^*) - \gamma\hat{x^*}\mathbf{b}(\mathbf{g},\lambda^*) + \theta\mathbf{b}_\delta(\mathbf{g},\lambda^*)\end{aligned}$$

which is equivalent to:

$$\beta\mathbf{x}^* = \alpha\mathbf{b}(\mathbf{g},\lambda^*) - \gamma\mathbf{x}^*\hat{b}(\mathbf{g},\lambda^*) + \theta\mathbf{b}_\delta(\mathbf{g},\lambda^*)$$

where $\hat{b}(\mathbf{g},\lambda^*) = \sum_{i=1}^n b_i(\mathbf{g},\lambda^*)$. By rearranging terms we get the following[24]:

$$\mathbf{x}^*(\boldsymbol{\Sigma}) = \frac{\alpha\mathbf{b}(\mathbf{g},\lambda^*) + \theta\mathbf{b}_\delta(\mathbf{g},\lambda^*)}{\beta + \gamma\hat{b}(\mathbf{g},\lambda^*)}$$

Given that $\alpha + \theta\delta > 0$ and $b_i(\mathbf{g},\lambda^*) + b_\delta(\mathbf{g},\lambda^*) \geq 1$ for all $i = 1, ..., n$, there is only one critical point and $\frac{\partial^2 U_i}{\partial x_i^2} = \sigma_{ii} < 0$ is always concave. This argument ensures that x^* is interior. Now, we establish uniqueness by dealing with the corner solutions.

Let $\beta(\boldsymbol{\Sigma}), \gamma(\boldsymbol{\Sigma}), \lambda(\boldsymbol{\Sigma})$ and $\mathbf{G}(\boldsymbol{\Sigma})$ be the elements of the decomposition of $\boldsymbol{\Sigma}$. For all matrices $\mathbf{Y}$, vector $\mathbf{y}$ and set $S \subset 1, 2, ..., n$, $\mathbf{Y}_s$ is a submatrix of Y with s rows and columns and $\mathbf{y}_s$ is the subvector of $\mathbf{y}$ with rows in s. Then, $\gamma(\boldsymbol{\Sigma}_s) \leq \gamma(\boldsymbol{\Sigma})$, $\beta(\boldsymbol{\Sigma}_s) \geq \beta(\boldsymbol{\Sigma})$ and $\lambda(\boldsymbol{\Sigma}_s) \leq \lambda(\boldsymbol{\Sigma})$. Also, $\lambda(\mathbf{G}) = \boldsymbol{\Sigma} + \gamma(\mathbf{U} - \mathbf{I}) - \sigma_{ii}\mathbf{I} - \theta_z Z$ and the coefficients in $\lambda\mathbf{G}$ (s rows and columns) are at least as high as the coefficients in $\lambda(\boldsymbol{\Sigma}_s)\mathbf{G}_s$. From Theorem I of Debreu and Herstein (1953, pg.600), $\rho_1(\lambda(\boldsymbol{\Sigma}_s)\mathbf{G}_s) \leq \rho_1(\lambda(\boldsymbol{\Sigma})\mathbf{G})$. Therefore, $\beta(\boldsymbol{\Sigma}) > \lambda(\boldsymbol{\Sigma})\rho_1(\mathbf{G})$ implies that $\beta(\boldsymbol{\Sigma}_s) > \lambda(\boldsymbol{\Sigma}_s)\rho_1(\mathbf{G}_s)$.

Let $\boldsymbol{y^*}$ be a non interior Nash equilibrium of the Team Game. Let $S \subset 1, 2, ..., n$ such that $y_i^* = 0$ if and only if $i \in N \setminus S$. Thus, $y_i^* > 0$ for all $i \in S$.

$$\begin{aligned}&\frac{\partial U_i}{\partial x_i} = \alpha - \beta x_i - \gamma\sum_{j\neq i}^n x_j + \lambda\sum_{i=1}^n g_{ij}x_j + \theta\delta_i \\ &\frac{\partial U_i}{\partial x_i}(0) = \alpha_i + \theta\delta_i\end{aligned}$$

and 0 cannot be a Nash Equilibrium. Then,

$$\begin{aligned}&-\boldsymbol{\Sigma}_s\boldsymbol{y}_s^* = (\beta\mathbf{I}_s + \gamma\mathbf{U}_s - \lambda\mathbf{G}_s)\boldsymbol{y}_s^* = \boldsymbol{\alpha} + \theta\boldsymbol{\delta} \\ &\beta\boldsymbol{y}_s^* + \gamma\mathbf{U}_s\boldsymbol{y}_s^* - \lambda\mathbf{G}_s\boldsymbol{y}_s^* = \alpha + \theta\boldsymbol{\delta}_s \\ &\beta[\mathbf{I}_s - \lambda^*\mathbf{G}_s]\boldsymbol{y}_s^* = \alpha + \theta\boldsymbol{\delta}_s - \gamma\hat{y_s^*}\cdot\mathbf{1_s}\end{aligned}$$

[24]The last step of writing the Nash Equilibrium follows from the simple algebra stated in BCZ (2006) page 1414.

where the last step utilizes $\mathbf{U}_s\mathbf{y}_s^* = \hat{y_s^*}\cdot\mathbf{1_s}$ and $\lambda^* = \lambda/\beta$. Pre-multiplying both sides by $[\mathbf{I}_s - \lambda^*\mathbf{G}_s]^{-1}$ yields:

$$\beta \boldsymbol{y_s^*} = [\mathbf{I} - \lambda^*\mathbf{G}_s]^{-1}\alpha + \theta[\mathbf{I} - \lambda^*\mathbf{G}_s]^{-1}\boldsymbol{\delta_s} - \gamma\hat{y_s^*}[\mathbf{I}_s - \lambda^*\mathbf{G}_s]^{-1}\cdot\mathbf{1_s} \tag{13}$$

$$\boldsymbol{y_s^*} = \frac{(\alpha - \gamma\hat{y_s^*})\mathbf{b}_s(\mathbf{g},\lambda^*) + \theta\mathbf{b}_\delta^s(\mathbf{g},\lambda^*)}{\beta} \tag{14}$$

Every player $i \in N \setminus S$ is best responding with $y_i^* = 0$ so that y_j^* is the action of the subset S of players.

$$\frac{\partial U_i}{\partial x_i}(y_i^*) = \alpha - \sum_{j\in S}\sigma_{ij}y_j^* + \theta\delta_i$$
$$\frac{\partial U_i}{\partial x_i}(y_i^*) = \alpha - \gamma\hat{y_s^*} + \lambda\sum_{j\in S}g_{ij}y_j^* + \theta\delta_i \leq 0$$

for all $i \in N \setminus S$. Now substitute y_s^* instead of y_j^* in the above equation:

$$\frac{\partial U_i}{\partial x_i}(\mathbf{y}^*) = \alpha - \gamma\hat{y_s^*} + \lambda\sum_{j\in S}g_{ij}\left(\frac{(\alpha - \gamma\hat{y_s^*})b_j(\mathbf{g},\lambda^*) + \theta\mathbf{b}_\delta^j(\mathbf{g},\lambda^*)}{\beta}\right) \leq 0$$
$$\frac{\partial U_i}{\partial x_i}(\mathbf{y}^*) = (\alpha - \gamma\hat{y_s^*})[1 + \lambda^*\sum_{j\in S}g_{ij}\mathbf{b}_j(\mathbf{g},\lambda^*)] + \theta\lambda^*\sum_{j\in S}\mathbf{b}_\delta^j(\mathbf{g},\lambda^*) \leq 0$$

If $\theta \leq |\alpha - \gamma\hat{y_s^*}|$ then $y_i^* \leq 0$ using Equations (10) and (11), which is a contradiction. Note that to reestablish uniqueness θ has to be small enough such that $\theta \leq |\alpha - \gamma\hat{y_s^*}|$.

Proof of **Proposition 1.a**:

Note that equation (9) still holds for this case. $\mathbf{\Sigma}$ matrix is substituted in equation (10) to obtain:

$$(\beta\mathbf{I} + \gamma\mathbf{U} - \lambda\mathbf{G})x^* = \boldsymbol{\alpha} + \theta\boldsymbol{\delta}$$

where $\boldsymbol{\alpha}$ is now a $n\times 1$ column vector and its elements shows the returns to individual actions. Now, substitute $\mathbf{x}\cdot 1$ instead of $\mathbf{U}\cdot x^*$:

$$[\beta\mathbf{I} - \lambda\mathbf{G}]x^* = \boldsymbol{\alpha} - \gamma x^*\cdot\mathbf{1} + \theta\boldsymbol{\delta} \longrightarrow \beta[\mathbf{I} - \lambda^*\mathbf{G}]x^* = \boldsymbol{\alpha} - \gamma x^*\cdot 1 + \theta\boldsymbol{\delta}$$

And pre-multiply both sides by $[\mathbf{I} - \lambda^*\mathbf{G}]^{-1}$ matrix to obtain:

$$\beta x^* = [\mathbf{I} - \lambda^*\mathbf{G}]^{-1}(\boldsymbol{\alpha} + \theta\boldsymbol{\delta}) - \gamma x^*[\mathbf{I} - \lambda^*\mathbf{G}]^{-1}\cdot\mathbf{1}$$
$$\beta x^* = \mathbf{b}_\alpha(\mathbf{g},\lambda^*) + \theta\mathbf{b}_\delta + \gamma x^*\mathbf{b}(\mathbf{g},\lambda^*)$$
$$x^*(\Sigma) = \frac{\mathbf{b}_\alpha(\mathbf{g},\lambda^*) + \theta\mathbf{b}_\delta(\mathbf{g},\lambda^*)}{\beta + \gamma\mathbf{b}(\mathbf{g},\lambda^*)}$$

Proof of **Proposition 1.b**:

Define:

$$\widetilde{\alpha_i} = \frac{\alpha_i}{\sigma_{ii}} \quad , \quad \widetilde{\sigma_{ij}} = \frac{\sigma_{ij}}{\sigma_{ii}} \quad , \quad \widetilde{\delta_i} = \frac{\delta_i}{\sigma_{ii}}$$

Now, rewrite the payoff function by using the above definition such that:

$$U_i = \widetilde{\alpha} x_i + \frac{1}{2}\widetilde{\sigma}_{ii} x_i^2 + \sum_{j \neq i} \widetilde{\sigma}_{ij} x_i x_j + \theta \widetilde{Z}$$

$$U_i = \frac{\sigma_i}{|\sigma_{ii}|} x_i + \frac{1}{2}\frac{\sigma_{ii}}{|\sigma_{ii}|} x_{ii}^2 + \sum_{j \neq i} \frac{\sigma_{ij}}{|\sigma_{ii}|} x_i x_j + \theta \widetilde{Z}$$

$$\frac{\partial U_i}{\partial x_i} = \frac{\alpha_i}{|\sigma_{ii}|} + \frac{\sigma_{ii}}{|\sigma_{ii}|} x_i + \sum_{j \neq i} \frac{\sigma_{ij}}{|\sigma_{ii}|} x_j + \theta \frac{\delta_i}{|\sigma_{ii}|} = 0$$

$$\frac{\partial U_i}{\partial x_i} = \frac{1}{|\sigma_{ii}|}(\alpha_i + \sigma_{ii} x_i + \sum_{j \neq i} x_j + \theta \delta_i) = 0$$

$$\frac{1}{|\sigma_{ii}|}(\alpha_i + \sigma_{i1} x_1 + \sigma_{i2}, x_2 + ... + \sigma_{in} x_n + \theta \delta_i) = 0 \quad \forall\ i = 1, \ldots, n$$

Let $\widetilde{\boldsymbol{\Sigma}}$ be the following matrix:

$$\begin{bmatrix} \frac{1}{\sigma_{11}} & 0 & 0 & \cdots & 0 \\ 0 & \frac{1}{\sigma_{22}} & 0 & \cdots & 0 \\ 0 & 0 & \frac{1}{\sigma_{33}} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & \frac{1}{\sigma_{nn}} \end{bmatrix}$$

Then,

$$\widetilde{\boldsymbol{\Sigma}}(\boldsymbol{\alpha} + \boldsymbol{\Sigma}\mathbf{x} + \theta\boldsymbol{\delta}) = \mathbf{0}$$

In the above equation, $\widetilde{\boldsymbol{\Sigma}}$ is not zero matrix (since the diagonal elements are not equal to 0), then the second term must be equal to a zero vector. By using this, if the equation is solved for $\mathbf{x}$, then:

$$\mathbf{x}^*(\boldsymbol{\Sigma}) = \frac{\mathbf{b}_{\widetilde{\alpha}}(\mathbf{g}, \widetilde{\lambda}^*) + \theta \widetilde{\mathbf{b}_{\delta}}(\mathbf{g}, \lambda^*)}{\widetilde{\beta} + \widetilde{\gamma}\hat{b}(\mathbf{g}, \widetilde{\lambda}^*)} \tag{15}$$

Proof of **Theorem 2.2**

Aggregate Nash equilibrium in the Team Game depends on the Bonacich centrality and the Bonacich centrality weighted by the ability parameter of the player. Note that $\rho_1(\mathbf{G}) > \rho_1(\mathbf{G}^{-i})$. Thus, when $\mathbf{M}(\mathbf{g}, \lambda^*)$ is well defined and nonnegative then so is $\mathbf{M}(\mathbf{g}^{-i}, \lambda^*)$ for all $i = 1, ..., n$.

Let $b_{ji}(\mathbf{g}, \lambda^*) = b_j(\mathbf{g}, \lambda^*) - b_j(\mathbf{g}^{-i}, \lambda^*)$ and $b_{ji}^{\delta}(\mathbf{g}, \lambda^*) = b_j^{\delta}(\mathbf{g}, \lambda^*) - b_j^{\delta}(\mathbf{g}^{-i}, \lambda^*)$ for $j \neq i$ which is the contribution of player i to player j's Bonacich centrality and Bonacich centrality weighted with the ability parameter respectively. The removal of player i from the network has the following effect:

$$\mathbf{b}(\mathbf{g},\lambda^*) - \mathbf{b}(\mathbf{g}^{-i},\lambda^*) + \mathbf{b}^{\delta}(\mathbf{g},\lambda^*) - \mathbf{b}^{\delta}(\mathbf{g}^{-i},\lambda^*) \equiv d_i(\mathbf{g},\lambda^*)$$

where d_i is the loss function when player i is removed from the network. Our goal is to find i^{th} player whose removal will result in the highest d_i such that $d_{i^*}(\mathbf{g},\lambda^*) \geq d_i(\mathbf{g},\lambda^*)$ for all $i = 1, ..., n$. The solution of the first two terms is given by BCZ (2006) on page 1412 under Remark 5, so we focus on the last two terms coming from the additional terms in the team game.

$$\begin{aligned}
\mathbf{b}^{\delta}(\mathbf{g},\lambda^*) - \mathbf{b}^{\delta}(\mathbf{g}^{-i},\lambda^*) &= b_i^{\delta}(\mathbf{g},\lambda^*) + \sum_{j\neq i}(\mathbf{b}_j^{\delta}(\mathbf{g},\lambda^*) - \mathbf{b}_j^{\delta}(\mathbf{g}^{-i},\lambda^*) \\
&= b_i^{\delta}(\mathbf{g},\lambda^*) + \sum_{j\neq i}\sum_{k=1}^{n}(m_{jk}(\mathbf{g},\lambda^*) - m_{jk}(\mathbf{g},\lambda^*)\delta_j
\end{aligned}$$

Lemma 1: Let $\mathbf{M} = [\mathbf{I} - a\mathbf{G}]^{-1}$ matrix be well defined and nonnegative. Then $m_{ji}(\mathbf{g},a)m_{ik}(\mathbf{g},a) = m_{ii}(\mathbf{g},a)[m_{jk}(\mathbf{g},a) - m_{jk}(\mathbf{g}^{-i},a)]$ for all $k \neq i \neq j$.

From Lemma 1 it follows that

$$\mathbf{b}^{\delta}(\mathbf{g},\lambda^*) - \mathbf{b}^{\delta}(\mathbf{g}^{-i},\lambda^*) = b_i^{\delta}(\mathbf{g},\lambda^*)\sum_{j\neq i}\sum_{k=1}^{n}\frac{m_{ji}(\mathbf{g},\lambda^*)m_{ik}(\mathbf{g},\lambda^*)}{m_{ii}(\mathbf{g},\lambda^*)}\delta_j$$

which can be rewritten as:

$$\begin{aligned}
\mathbf{b}^{\delta}(\mathbf{g},\lambda^*) - \mathbf{b}^{\delta}(\mathbf{g}^{-i},\lambda^*) &= \sum_{k=1}^{n} m_{ik}(\mathbf{g},\lambda^*)\delta_i + \sum_{k=1}^{n}\sum_{j\neq i}\frac{m_{ji}(\mathbf{g},\lambda^*)m_{ik}(\mathbf{g},\lambda^*)}{m_{ii}(\mathbf{g},\lambda^*)}\delta_j \\
&= \sum_{k=1}^{n}(m_{ik}(\mathbf{g},\lambda^*)\delta_i + \sum_{j\neq i}\frac{m_{ji}(\mathbf{g},\lambda^*)m_{ik}(\mathbf{g},\lambda^*)}{m_{ii}(\mathbf{g},\lambda^*)}\delta_j) \\
&= \sum_{k=1}^{n}(m_{ik}(\mathbf{g},\lambda^*)\delta_i + \frac{m_{ik}(\mathbf{g},\lambda^*)}{m_{ii}(\mathbf{g},\lambda^*)}\sum_{j\neq i} m_{ji}\delta_j)
\end{aligned}$$

By rearranging terms we obtain:

$$\mathbf{b}^{\delta}(\mathbf{g},\lambda^*) - \mathbf{b}^{\delta}(\mathbf{g}^{-i},\lambda^*) = \sum_{k=1}^{n} m_{ik}(\mathbf{g},\lambda^*)(\delta_i + \frac{\sum_{j\neq i} m_{ji}(\mathbf{g},\lambda^*)\delta_j}{m_{ii}(\mathbf{g},\lambda^*)})$$

Using the definition $b_i(\mathbf{g},\lambda^*) = \sum_{k=1}^{n} m_{ik}(\mathbf{g},\lambda^*)$, we obtain:

$$\mathbf{b}^{\delta}(\mathbf{g},\lambda^*) - \mathbf{b}^{\delta}(\mathbf{g}^{-i},\lambda^*) = b_i(\mathbf{g},\lambda^*)(\delta_i + \frac{\sum_{j\neq i} m_{ji}(\mathbf{g},\lambda^*)\delta_j}{m_{ii}(\mathbf{g},\lambda^*)})$$

The above expression measures the effect of player i's removal Bonacich centrality weighted with ability parameter. Combining the above expression with the ICM to get the full effect of player i's removal ($d_i(\mathbf{g}, \lambda^*)$) yields:

$$b_i(\mathbf{g}, \lambda^*) \times \frac{\sum_{j=1}^{n} m_{ji}(\mathbf{g}, \lambda^*)}{m_{ii}(\mathbf{g}, \lambda^*)} + b_i(\mathbf{g}, \lambda^*) \times (\delta_i + \frac{\sum_{j \neq i} m_{ji}(\mathbf{g}, \lambda^*)\delta_j}{m_{ii}(\mathbf{g}, \lambda^*)})$$

By taking into $b_i(\mathbf{g}, \lambda^*)$ parentheses:

$$b_i(\mathbf{g}, \lambda^*) \times (\frac{\sum_{j=1}^{n} m_{ji}(\mathbf{g}, \lambda^*) + \sum_{j \neq i} m_{ji}(\mathbf{g}, \lambda^*)\delta_j}{m_{ii}(\mathbf{g}, \lambda^*)} + \delta_i)$$

The above expression can be further simplified as:

$$\bar{c}_i(\mathbf{g}, a) = b_i(\mathbf{g}, \lambda^*) \times (\frac{\sum_{j=1}^{n} m_{ji}(\mathbf{g}, \lambda^*) + \sum_{j=1}^{n} m_{ji}(\mathbf{g}, \lambda^*)\delta_j}{m_{ii}(\mathbf{g}, \lambda^*)})$$

Proof of **Theorem 2.3**

Nash Equilibrium of the team game does not take into account the externality that a player gets from her teammates. It might be the case that social planner is interested in taking the externality into account as well as considering the each player's effect on the interaction matrix. Therefore, the loss function (the effect of player i's removal) becomes:

$$b(\mathbf{g}, \lambda^*) - b(\mathbf{g}^{-i}, \lambda^*) + r_\delta^i(\mathbf{g}, \lambda^*) = b_i(\mathbf{g}, \lambda^*) + \sum_{j \neq i} b_{ji}(\mathbf{g}, \lambda^*) + r_\delta^i(\mathbf{g}, \lambda^*) \equiv e_i(\mathbf{g}, \lambda^*)$$

where e_i is the loss function when player i is removed from the network. Our goal is to find i^{th} player whose removal will result in the highest e_i such that $e_{i^*}(\mathbf{g}, \lambda^*) \geq e_i(\mathbf{g}, \lambda^*)$ for all $i = 1, ..., n$.. By following this approach, we come up with $TICM^e$ which is the following:

$$\hat{c}_i(\mathbf{g}, a) = b_i(\mathbf{g}, a) \times \frac{\sum_{j=1}^{n} m_{ji}(\mathbf{g}, a)}{m_{ii}(\mathbf{g}, a)} + \sum_{j=1}^{n} m_{ji}(\mathbf{g}, a) \times \delta_i$$

8 Tables

Table 1: Spain vs Germany Final Game, a=0.125 and d=0.5

Name	Position	$\hat{c}_i$	c_i	Name	Position	$\hat{c}_i$	c_i
Xavi	M	4.31	3.97	Lahm	D	4.38	3.73
Fabregas	M	3.97	3.35	Schweinsteiger	M	4.37	3.57
Senna	M	3.66	3.66	Frings	M	4.02	4.02
Ramos	D	3.53	3.53	Podolski	M	4.02	3.45
Capdevila	D	3.49	3.49	Metzelder	D	3.88	3.88
Puyol	D	3.48	3.48	Mertesacker	D	3.77	3.77
Silva	M	3.47	3.36	Ballack	M	3.73	3.38
Guiza*	F	3.46	3.02	Klose	F	3.57	3.14
Marchena	D	3.46	3.46	Hitzlsperger	M	3.57	3.57
Iniesta	M	3.45	3.45	Friedrich	D	3.47	3.47
Torres	F	3.18	2.98	Lehmann	G	3.34	3.34
Xabi Alonso*	M	3.14	3.14	Jansen*	M	3.25	3.25
Cazorla*	M	3.11	3.11	Gomez*	F	3.12	3.12
Casillas	G	3.07	3.07	Kuranyi*	F	2.99	2.99

Table 1: In the above Table, the first 4 columns are for Spain and the remaining ones are for Germany. $\hat{c}_i$ represents $TICM^e$ and c_i represents ICM. * indicates that player is a substitute.

Table 2: Spain vs Russia Semi-Final Game, a=0.125 and d=0.5

Name	Position	$\hat{c}_i$	c_i	Name	Position	$\hat{c}_i$	c_i
Fabregas*	M	6.04	5.24	Zyryanov	M	4.87	4.56
Silva	M	5.50	5.35	Semak	M	4.37	4.37
Xavi	M	5.48	5.09	Zhirkov	D	4.12	4.12
Ramos	D	5.37	5.37	Anyukov	D	4.10	4.10
Iniesta	D	4.97	4.97	Arshavin	F	4.06	3.70
Senna	M	4.82	4.82	Ignashevich	D	3.84	3.84
Torres	F	4.30	4.06	Berezutski	D	3.83	3.83
Capdevila	D	4.23	4.23	Saenko	M	3.63	3.63
Xabi Alonso*	M	4.11	4.11	Semshov	M	3.57	3.57
Villa	F	4.08	3.59	Pavlyuchenko	F	3.52	3.30
Guiza*	F	4.06	3.58	Sychev*	F	3.46	3.46
Marchena	D	4.02	4.02	Akinfeev	G	3.36	3.36
Puyol	D	3.75	3.75	Biyaletdinov*	M	3.28	3.28
Casillas	G	3.69	3.69				

Table 2: In the above Table, the first 4 columns are for Spain and the remaining ones are for Russia. $\hat{c}_i$ represents $TICM^e$ and c_i represents ICM. * indicates that player is a substitute.

Table 3: Germany vs Turkey Semi-Final Game, a=0.125 and d=0.5

Name	Position	$\hat{c}_i$	c_i	Name	Position	$\hat{c}_i$	c_i
Schweinsteiger	M	5.01	4.15	Ugur	M	4.95	4.23
Lahm	D	4.48	3.82	Hamit	M	4.73	4.73
Podolski	M	4.12	3.55	Semih	F	4.69	3.85
Mertesacker	D	3.85	3.85	Ayhan	M	4.65	4.65
Friedrich	D	3.75	3.75	Sabri	D	4.64	4.64
Hitzlsperger	M	3.75	3.75	Hakan	D	4.32	4.32
Frings*	M	3.68	3.68	Kazim	M	4.23	4.23
Metzelder	D	3.67	3.67	Aurelio	M	4.15	4.15
Ballack	M	3.63	3.29	Gokhan	D	4.14	4.14
Klose	F	3.51	3.07	Mehmet	M	4.11	4.11
Rolfes	M	3.22	3.22	Gokdeniz*	M	3.45	3.45
Lehmann	G	2.98	2.98	Rustu	G	3.44	3.44
Jansen*	M	2.91	2.91	Mevlut	F	3.43	3.43
				Tumer*	M	3.34	3.34

Table 3: In the above Table, the first 4 columns are for Germany and the remaining ones are for Turkey. $\hat{c}_i$ represents $TICM^e$ and c_i represents ICM. * indicates that player is a substitute.

Table 4: Netherlands vs Russia Quarter Final Game, a=0.125 and d=0.5

Name	Position	$\hat{c}_i$	c_i	Name	Position	$\hat{c}_i$	c_i
Van Bronchorst	D	5.40	4.34	Arshavin	F	5.04	4.61
Van Der Vaart	M	5.08	5.08	Zhirkov	D	4.56	4.56
Nistelrooy	F	5.08	4.63	Pavlyuchenko	F	4.47	4.24
Sneijder	M	4.51	4.30	Zyryanov	M	4.45	4.16
Van Persie*	F	4.51	4.16	Semak	M	4.22	4.22
Heitinga*	D	4.43	4.43	Torbinski*	M	4.16	3.68
Boulahrouz	D	4.39	4.39	Anyukov	D	4.01	4.01
Oojer	D	4.35	4.35	Semshov	M	3.91	3.91
De Jong	M	4.24	4.24	Saenko	M	3.90	3.90
Kuyt	M	4.24	3.73	Kolodin	D	3.88	3.88
Afellay*	M	4.16	4.16	Ignashevich	D	3.76	3.76
Van Der Sarr	G	4.09	4.09	Bilyaletdinov*	M	3.68	3.68
Englaar	M	4.03	4.03	Akinfeev	G	3.52	3.52
Mathijsen	D	4.00	4.00	Sychev*	F	3.40	3.40

Table 4: In the above Table, the first 4 columns are for Netherlands and the remaining ones are for Russia. $\hat{c}_i$ represents $TICM^e$ and c_i represents ICM. * indicates that player is a substitute.

Table 5: Germany vs Portugal Quarter Final Game, a=0.125 and d=0.5

Name	Position	$\hat{c}_i$	c_i	Name	Position	$\hat{c}_i$	c_i
Schweinsteiger	M	5.47	4.58	Deco	M	6.44	5.81
Podolski	M	5.20	4.55	Ronaldo	M	5.47	5.31
Ballack	M	5.10	4.69	Simao	M	5.39	5.39
Klose	F	5.01	3.96	Bosingwa	D	5.18	5.18
Lahm	D	4.95	4.25	Pepe	D	5.15	4.63
Rolfes	M	4.38	4.38	Meireles*	M	4.91	4.48
Hitzlsperger	M	4.05	4.05	Ferreira	D	4.70	4.70
Friedrich	D	3.87	3.87	Petit	M	4.51	4.51
Lehmann	G	3.60	3.60	Carvalho	D	4.39	4.39
Mertesacker	D	3.58	3.58	Nuno Gomes	F	4.39	3.98
Metzelder	D	3.57	3.57	Postiga*	F	4.26	3.85
Fritz*	M	3.39	3.39	Moutinho	M	4.02	4.02
Borowski*	M	3.32	3.32	Nani*	M	4.02	4.02
Jansen*	M	3.31	3.31	Ricardo	G	3.81	3.81

Table 5: In the above Table, the first 4 columns are for Germany and the remaining ones are for Portugal. $\hat{c}_i$ represents $TICM^e$ and c_i represents ICM. * indicates that player is a substitute.

Table 6: Spain vs Italy Quarter Final Game, a=0.125 and d=0.5

Name	Position	$\hat{c}_i$	c_i	Name	Position	$\hat{c}_i$	c_i
Fabregas*	M	8.43	7.49	Grosso	D	5.74	5.74
Silva	M	8.15	7.97	De Rossi	M	5.13	5.13
Xavi	M	7.59	7.13	Panucci	D	4.99	4.29
Capdevila	D	7.53	7.53	Ambrossini	M	4.93	4.93
Senna	M	7.23	7.23	Aquilani	M	4.73	4.73
Villa	F	6.66	6.00	Zambrotta	D	4.65	4.65
Ramos	D	6.52	6.52	Camoranesi*	M	4.65	4.65
Marchena	D	6.14	6.14	Chiellini	D	4.56	4.56
Iniesta	M	5.67	5.67	Toni	F	4.38	4.38
Puyol	D	5.63	5.63	Cassano	F	4.28	4.28
Torres	F	5.56	5.29	Buffon	G	4.05	4.05
Guiza*	F	5.33	4.77	Di Natale*	F	4.05	4.05
Cazorla*	M	5.18	5.18	Perrotta	M	3.94	3.94
Casillas	G	4.72	4.72	Del Piero*	F	3.59	3.59

Table 6: In the above Table, the first 4 columns are for Spain and the remaining ones are for Italy. $\hat{c}_i$ represents $TICM^e$ and c_i represents ICM. * indicates that player is a substitute.

Table 7: Croatia vs Turkey Quarter Final Game, a=0.125 and d=0.5

Name	**Position**	$\hat{c}_i$	c_i	**Name**	**Position**	$\hat{c}_i$	c_i
Modric	M	5.69	4.58	Arda	M	7.67	5.26
Pranjic	D	4.47	4.47	Hamit	M	5.24	5.24
Rakitic	M	4.18	4.18	Tuncay	M	5.24	5.24
Srna	M	4.06	3.83	Hakan	D	5.11	5.11
Klasnic*	F	4.05	3.31	Nihat	F	5.03	4.40
N. Kovac	M	3.97	3.97	Semih*	F	4.89	3.99
Simunic	D	3.90	3.90	Ugur*	M	4.48	3.82
Corluka	D	3.77	3.77	Sabri	D	4.24	4.24
R. Kovac	D	3.77	3.77	Gokhan	D	4.14	4.14
Kranjcar	M	3.66	3.66	Emre	D	4.13	4.13
Olic	F	3.64	3.37	Mehmet	M	4.09	4.09
Petric*	F	3.32	3.32	Kazim	M	3.91	3.91
Pletikosa	G	3.21	3.21	Rustu	G	3.69	3.69
				Gokdeniz*	M	3.43	3.43

Table 7: In the above Table, the first 4 columns are for Croatia and the remaining ones are for Turkey. $\hat{c}_i$ represents $TICM^e$ and c_i represents ICM. * indicates that player is a substitute.

Table 8: Spain vs Germany Final Game, $TICM$, $TICM^e$ and ICM a=0.125 and d=0.5

Name	**Position**	$\bar{c}_i$	$\hat{c}_i$	$\tilde{c}_i$	**Name**	**Position**	$\bar{c}_i$	$\hat{c}_i$	c_i
Xavi	M	4.42	4.31	3.97	Lahm	D	4.53	4.38	3.73
Fabregas	M	4.04	3.97	3.35	Schweinsteiger	M	4.45	4.31	3.57
Senna	M	3.78	3.66	3.66	Frings	M	4.25	4.02	4.02
Ramos	D	3.64	3.53	3.53	Podolski	F	4.15	4.01	3.45
Capdevila	D	3.60	3.49	3.49	Metzelder	D	4.07	3.88	3.88
Puyol	D	3.59	3.48	3.48	Mertesacker	D	3.95	3.77	3.77
Marchena	D	3.57	3.46	3.46	Ballack	M	3.88	3.71	3.38
Silva	M	3.57	3.47	3.36	Hitzspelger	M	3.77	3.58	3.58
Iniesta	M	3.57	3.45	3.45	Klose	F	3.71	3.59	3.13
Guiza	F	3.53	3.46	3.02	Friedrich	D	3.64	3.47	3.47
Torres	F	3.26	3.18	2.98	Lehmann	G	3.50	3.34	3.34
Xabi	M	3.24	3.14	3.14	Jansen	D	3.41	3.25	3.25
Santi	M	3.21	3.11	3.11	Gomez	F	3.27	3.12	3.12
Casillas	G	3.16	3.07	3.07	Kuranyi	F	3.12	2.99	2.99

Table 8: In the above Table, the first 5 columns are for Spain and the remaining ones are for Germany. $\bar{c}_i$ represents $TICM$, $\hat{c}_i$ represents $TICM^e$, and c_i represents ICM. * indicates that player is a substitute.

Table 9: Netherlands vs Russia Quarter Final Game, $TICM$ and $TICM^e$ a=0.125 and d=0.5

Name	**Position**	$\bar{c_i}$	$\hat{c_i}$	$\tilde{c_i}$	**Name**	**Position**	$\bar{c_i}$	$\hat{c_i}$	c_i
Gio	D	5.50	5.32	4.34	Arshavin	F	5.16	4.98	4.61
Vaart	M	5.31	5.08	5.08	Zhirkov	D	4.68	4.56	4.59
Nistelrooy	F	5.29	5.05	4.63	Pavlyuchenko	F	4.61	4.47	4.24
Sneijder	M	4.68	4.51	4.30	Zyryanov	M	4.53	4.46	4.16
Van Persie	F	4.68	4.50	4.16	Semak	M	4.33	4.22	4.22
Boulahrouz	D	4.64	4.39	4.39	Torbinski	M	4.22	4.17	3.68
Heitinga	D	4.63	4.43	4.43	Anyukov	D	4.11	4.01	4.01
Ooijer	D	4.53	4.35	4.35	Semshov	M	4.01	3.91	3.91
De Jong	M	4.41	4.24	4.24	Saenko	M	4.01	3.90	3.90
Afellay	M	4.38	4.16	4.16	Kolodin	D	3.97	3.88	3.88
Kuyt	F	4.37	4.21	3.73	Ignashevich	D	3.85	3.76	3.76
Van der Sarr	G	4.25	4.09	4.09	Bilyaletdinov	M	3.77	3.68	3.68
Engelaar	M	4.21	4.03	4.03	Akinfeev	G	3.60	3.52	3.52
Mathijsen	D	4.19	4.00	4.00	Sychev	F	3.48	3.40	3.40

Table 9: In the above Table, the first 5 columns are for Netherlands and the remaining ones are for Russia. $\bar{c_i}$ represents $TICM$, $\hat{c_i}$ represents $TICM^e$, and c_i represents ICM. * indicates that player is a substitute.

Table 10: Key Group of Spain in Euro 2008, $TICM^e$, a=0.125, d=0.5

Match	**Group Size**	**Player Position**	**Rank in** $\hat{c}$	**Player Names**	$\hat{c_g}$
Final	2	M, M	1,2	Xavi, Fabregas	7.78
Final	2	M, F	1,8	Xavi, Guiza	7.36
Final	3	M, M, F	1,2,8	Xavi, Fabregas, Guiza	10.58
Final	3	M, M, M	1,2,3	Xavi, Fabregas, Senna	10.49
Semi-Final	2	M, M	1,3	Fabregas, Xavi	10.48
Semi-Final	2	M, M	1,2	Fabregas, Silva	10.40
Semi-Final	3	M, M, M	1,2,3	Fabregas, Silva, Xavi	14.03
Semi-Final	3	M, M, D	1,2,4	Fabregas, Silva, Ramos	13.76
Quarter Final	2	M, M	1,3	Fabregas, Xavi	14.49
Quarter Final	2	M, M	1,2	Fabregas, Silva	14.15
Quarter Final	3	M, M, M	1,2,3	Fabregas, Silva, Xavi	18.87
Quarter Final	3	M, M, F	1,3,6	Fabregas, Xavi, Vila	18.66

Table 11: Key Group of Germany in Euro 2008, $TICM^e$, a=0.125, d=0.5

Match	**Group Size**	**Player Position**	**Rank in** $\hat{c}$	**Player Names**	$\hat{c_g}$
Final	2	D, M	1,2	Lahm, Schweinsteiger	8.28
Final	2	M, M	2,4	Schweinsteiger, Podolski	7.97
Final	3	D, M, M	1,2,4	Lahm, Schweinsteiger, Podolski	11.44
Final	3	M, M, M	2,3,4	Schweinsteiger, Frings, Podolski	11.25
Semi-Final	2	M, D	1,2	Schweinsteiger, Lahm	8.92
Semi-Final	2	M, M	1,3	Schweinsteiger, Podolski	8.52
Semi-Final	3	M, D, M	1,2,3	Schweinsteiger, Lahm, Podolski	11.93
Semi-Final	3	M, D, M	1,2,9	Schweinsteiger, Lahm, Ballack	11.69
Quarter Final	2	M, M	1,2	Schweinsteiger, Podolski	9.86
Quarter Final	2	M, F	1,4	Schweinsteiger, Klose	9.85
Quarter Final	3	M, M, F	1,2,4	Schweinsteiger, Podolski, Klose	13.71
Quarter Final	3	M, M, D	1,4,5	Schweinsteiger, Klose, Lahm	13.69

Table 12: Key Group of Russia in Euro 2008 $TICM^e$, a=0.125, d=0.5

Match	**Group Size**	**Player Position**	**Rank in $\hat{c}$**	**Player Names**	$\hat{c_g}$
Semi-Final	2	M, M	1,2	Zyryanov, Semak	8.36
Semi-Final	2	M, D	1, 4	Zyryanov, Anyukov	8.19
Semi-Final	3	M, M, F	1,2,5	Zyryanov, Semak, Arshavin	11.23
Semi-Final	3	M, D, F	1,4,5	Zyryanov, Anyukov, Arshavin	11.11
Quarter Final	2	F, M	1,4	Arshavin, Zyryanov	8.80
Quarter Final	2	F, F	1,3	Arshavin, Pavlyuchenko	8.79
Quarter Final	3	F, F, M	1,3,6	Arshavin, Pavlyuchenko, Torbinski	12.08
Quarter Final	3	F, D, F	1,2,3	Arshavin, Zhirkov, Pavlyuchenko	11.95

Table 13: Key Group of Turkey in Euro 2008 $TICM^e$, a=0.125, d=0.5

Match	**Group Size**	**Player Position**	**Rank in $\hat{c}$**	**Player Names**	$\hat{c_g}$
Semi-Final	2	M, F	1,2	Ugur, Semih	9.18
Semi-Final	2	F, D	2,5	Semih, Sabri	8.96
Semi-Final	3	M, F, D	1,2,5	Ugur, Semih, Sabri	12.60
Semi-Final	3	M, F, M	1,2,3	Ugur, Semih, Hamit	12.54
Quarter Final	2	M, F	1,5	Arda, Nihat	11.89
Quarter Final	2	M, F	1, 6	Arda, Semih	11.85
Quarter Final	3	M, F, F	1,5,6	Arda, Nihat, Semih	12.31
Quarter Final	3	M, M, F	1,2,6	Arda, Hamit, Semih	12.18

Table 14: Key Groups of Other Countries in Euro 2008 $TICM^e$, a=0.125, d=0.5

Match	**Group Size**	**Player Position**	**Rank in $\hat{c}$**	**Player Names**	$\hat{c_g}$
			Netherlands		
Quarter Final	2	D, F	1,3	Bronckhorst, Nistelrooy	9.81
Quarter Final	2	D, M	1,2	Bronckhorst, Vaart	9.64
Quarter Final	3	D, M, F	1,2,3	Bronckhorst, Vaart, Nistelrooy	13.27
Quarter Final	3	D, F, F	1,3,5	Bronckhorst, Nistelrooy, Persie	13.15
			Portugal		
Quarter Final	2	M, D	1,5	Deco, Pepe	10.73
Quarter Final	2	M, M	1,2	Deco, Ronaldo	10.60
Quarter Final	3	M, M, D	1,2,5	Deco, Ronaldo, Pepe	14.35
Quarter Final	3	M, D, M	1,5,6	Deco, Pepe, Meireles	14.15
			Italy		
Quarter Final	2	D, D	1,3	Grosso, Panucci	9.91
Quarter Final	2	D, M	1,2	Grosso, De Rossi	9.78
Quarter Final	3	D, M, D	1,2,3	Grosso, De Rossi, Panucci	13.47
Quarter Final	3	D, D, D	1,3,7	Grosso, Panucci, Zambrotta	13.25
			Croatia		
Quarter Final	2	F, M	1,5	Modric, Klasnic	9.21
Quarter Final	2	D, M	1,2	Modric, Pranjic	9.17
Quarter Final	3	D, F, M	1,2,5	Modric, Pranjic, Klasnic	12.31
Quarter Final	3	D, D, M	1,3,5	Modric, Rakitic, Klasnic	12.18

Table 15: Key Group of Spain in Euro 2008 ICM, a=0.125, d=0.5

Match	**Group Size**	**Player Position**	**Rank in $\tilde{c}$**	**Player Names**	$\tilde{c_g}$
Final	2	M, M	1, 2	Xavi, Senna	7.02
Final	2	M, D	1, 3	Xavi, Ramos	6.97
Final	3	M, D, D	1, 3, 4	Xavi, Ramos, Capdevila	9.61
Final	3	M, D, D	1, 3, 5	Xavi, Ramos, Puyol	9.60
Semi-Final	2	D, M	1,2	Ramos, Silva	9.52
Semi-Final	2	M, M	2,3	Silva, Fabregas	9.45
Semi-Final	3	M, D, M	5,1,3	Iniesta, Ramos, Fabregas	12.82
Semi-Final	3	M, M, M	5,4,8	Iniesta, Xavi, Xabi	12.73
Quarter Final	2	D, M	2,1	Capdevila, Silva	13.51
Quarter Final	2	M, M	4,1	Senna, Silva	13.13
Quarter Final	3	D, M, M	2,4,1	Capdevila, Senna, Silva	17.48
Quarter Final	3	D, M, M	2,4,3	Capdevila, Senna, Fabregas	17.39

Table 16: Descriptive Statistics

Variable	**Obs**	**Mean**	**Std. Dev**	**Min**	**Max**
Average Rating	113	6.17	0.85	3.83	8.17
ICM	113	4.21	0.61	2.99	5.81
$TICM^e$	113	4.38	0.74	2.99	7.67
Log Market Value	112	2.25	0.84	-0.22	4.32
Age	113	28.29	3.55	22	36
Height(cm)	112	181.27	6.85	168	198
Club UEFA Points	112	66.95	33.33	0	124.99
Nation UEFA Points	112	44.27	17.05	11.62	75.27
International Caps	112	46.52	21.53	10	98
International Goals	112	7.73	9.56	0.00	48
Captaincy	112	0.16	0.37	0.00	1.00
Right-footed	112	0.57	0.50	0.00	1.00
Left-footed	112	0.20	0.40	0.00	1.00
Both	112	0.23	0.42	0.00	1.00
Defender	113	0.27	0.45	0.00	1.00
Midfielder	113	0.47	0.50	0.00	1.00
Forward	113	0.25	0.43	0.00	1.00
Expiring	112	0.14	0.35	0.00	1.00
Move	112	0.38	0.49	0.00	1.00

Table 16: Average ratings for the players are obtained by taking the average of the player ratings available through Goal.com ESPN Soccer and Skysports. Log of the market value of players are obtained from *transfermarkt.de* along with the player characteristics. Club and Nation UEFA points are available from UEFA's website. We use 2007-2008 points, which is earned in 2003-2008 period by clubs or nations in UEFA tournaments. Captaincy is equal to 1 if the player is a captain in his club or his national team and 0 oth. D, M and F are dummy variables to indicate the field position of the players. They represent Defender, Midfielder and Forward positions respectively. Goalkeepers are excluded from the sample. Expiring is equal to 1 if the players current contract with his club is expiring at the end of 2009-2010 season 0 oth. Move is equal to 1 if the player transferred to another after Euro 2008 and 0 oth.

Table 17: Average Ratings, ICM and $TICM^e$ Pooled OLS Estimation

Variable	**I**	**II**	**III**	**IV**	**V**	**VI**
ICM	0.254***	0.248***	0.226			
	(0.093)	(0.094)	(0.146)			
$TICM^e$				0.308***	0.296***	0.262**
				(0.090)	(0.086)	(0.119)
ET	0.327*	0.375*	0.361*	0.303	0.345*	0.327
	(0.189)	(0.209)	(0.217)	(0.193)	(0.207)	(0.217)
Age	-0.198	-0.104	-0.135	-0.127	-0.036	-0.108
	(0.262)	(0.273)	(0.267)	(0.268)	(0.271)	(0.277)
Age Squared	0.003	0.001	0.002	0.002	0.000	0.002
	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)
Club UEFA pts	0.001	0.001	0.001	0.000	0.000	0.000
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
Nation UEFA pts	0.007*	0.008*	0.008*	0.008*	0.008*	0.008*
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)
D		-0.272*	-0.119		-0.233	0.052
		(0.149)	(0.935)		(0.145)	(0.836)
F		-0.066	-0.987		-0.099	-1.483
		(0.189)	(1.225)		(0.184)	(1.122)
DxI			-0.035			
			(0.213)			
FxI			0.236			
			(0.305)			
DxTI						-0.065
						(0.184)
FxTI						0.328
						(0.264)
Constant	7.886**	6.576*	7.122*	6.602*	5.374	6.564
	(3.807)	(3.928)	(3.864)	(3.895)	(3.957)	(4.083)
Observations	178	178	178	178	178	178
Number of Players	111	111	111	111	111	111
R-squared	0.119	0.131	0.137	0.153	0.162	0.176
Wald Chi Sq statistic	18.96	18.59	18.86	23.37	24.77	24.12

Table 17: The dependent variable is average ratings and the pooled OLS coefficients are reported in the above regressions. Bootstrapped robust standard errors are given in parentheses. ***, **, * indicate 1, 5 and 10 percent significance levels respectively. ET is a dummy variable which takes the value of 1 if the player played more than 90 minutes in any of the matches and 0 otherwise. Goalkeepers are excluded from the sample. DxICM, FxICM, Dx$TICM^e$ and Fx$TICM^e$ are interaction variables obtained by interacting the (team) intercentrality measure with the position dummy.

Table 18: Average Ratings and (Team) Intercentrality GLS Estimation

Variable	**I**	**II**	**III**	**IV**	**V**	**VI**
(ICM)	0.557***	0.564***	0.495**			
	(0.155)	(0.154)	(0.197)			
$(TICM^e)$				0.541***	0.534***	0.434***
				(0.146)	(0.135)	(0.165)
Age	-0.070	-0.066	-0.084	-0.034	0.058	-0.029
	(0.301)	(0.314)	(0.323)	(0.299)	(0.322)	(0.307)
Age Squared	0.001	0.001	0.001	-0.001	-0.001	0.000
	(0.005)	(0.005)	(0.006)	(0.005)	(0.006)	(0.005)
Club UEFA pts	-0.003	-0.003	-0.002	-0.004	-0.004	-0.004
	(0.003)	(0.003)	(0.004)	(0.003)	(0.003)	(0.003)
Nation UEFA pts	-0.001	0.001	0.001	0.001	0.001	0.001
	(0.005)	(0.006)	(0.007)	(0.005)	(0.005)	(0.005)
Defender		-0.066	-0.375		-0.001	-0.439
		(0.166)	(1.158)		(0.175)	(1.024)
Forward		0.015*	-1.067		-0.065	-2.189*
		(0.212)	(1.675)		(0.199)	(1.247)
D$\times ICM$			0.072			
			(0.280)			
F$\times ICM$			0.272			
			(0.418)			
D$\times TICM^e$						0.098
						(0.237)
F$\times TICM^e$						0.504*
						(0.296)
Constant	5.399	5.305	5.875	3.797	3.510	5.253
	(4.431)	(4.626)	(4.889)	(4.342)	(4.647)	(4.530)
Observations	112	112	112	112	112	112
R square	0.147	0.148	0.153	0.206	0.207	0.231
R square adj	0.106	0.091	0.078	0.169	0.154	0.163
Wald Chi Sq Statistic	14.29	14.83	18.18	15.51	16.76	26.63

Table 18: The dependent variable is average ratings and the GLS estimation results are reported in the above regressions. Bootstrapped robust standard errors with 1000 replications are given in parentheses. ***, **, * indicate 1, 5 and 10 percent significance levels respectively. Goalkeepers are excluded from the sample. D$\times ICM$, F$\times ICM$, D$\times TICM^e$ and F$\times TICM^e$ are interaction variables obtained by interacting the (team) intercentrality measure with the position dummy.

Table 19: Market Values and (Team) Intercentrality GLS Estimation

Variable	I	II	III	IV	V	VI
ICM	0.256**	0.311***	0.458***			
	(0.104)	(0.104)	(0.137)			
$TICM^e$				0.240***	0.247***	0.333***
				(0.082)	(0.079)	(0.103)
Age	0.430*	0.398	0.408*	0.474*	0.452*	0.505**
	(0.256)	(0.244)	(0.228)	(0.259)	(0.248)	(0.247)
Age squared	-0.010**	-0.009**	-0.009**	-0.010**	-0.010**	-0.011**
	(0.004)	(0.004)	(0.004)	(0.005)	(0.004)	(0.004)
Club UEFA pts	0.005**	0.005**	0.005**	0.005**	0.005**	0.006**
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
Nation UEFA pts	0.013***	0.012***	0.013***	0.014***	0.014***	0.014***
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)
Defender		-0.187	1.254		-0.166	0.597
		(0.135)	(1.028)		(0.133)	(0.992)
Forward		0.181	1.746		0.121	1.629*
		(0.153)	(1.053)		(0.147)	(0.931)
DxICM			-0.335			
			(0.233)			
FxICM			-0.387			
			(0.258)			
Dx$TICM^e$						-0.174
						(0.221)
Fx$TICM^e$						-0.357*
						(0.216)
Constant	-4.183	-3.982	-4.807	-4.825	-4.583	-5.779
	(3.739)	(3.520)	(3.317)	(3.750)	(3.604)	(3.570)
Observations	112	112	112	112	112	112
R square	0.512	0.536	0.551	0.523	0.509	0.552
R square adj	0.489	0.505	0.512	0.500	0.507	0.512
Wald Chi Sq Statistic	120.55	137.33	139.61	124.69	139.67	148.88

Table 19: The dependent variable is the natural log of 2010 market value of the players obtained from transfermarkt.de. Goalkeepers are excluded from the sample. DxICM, FxICM, Dx$TICM^e$ and Fx$TICM^e$ are variables obtained by interacting the (team) intercentrality measures with the position dummy of the player. Bootstrapped robust standard errors with 1000 replications are reported in parentheses. ***, **, * indicate 1, 5 and 10 percent significance levels respectively.